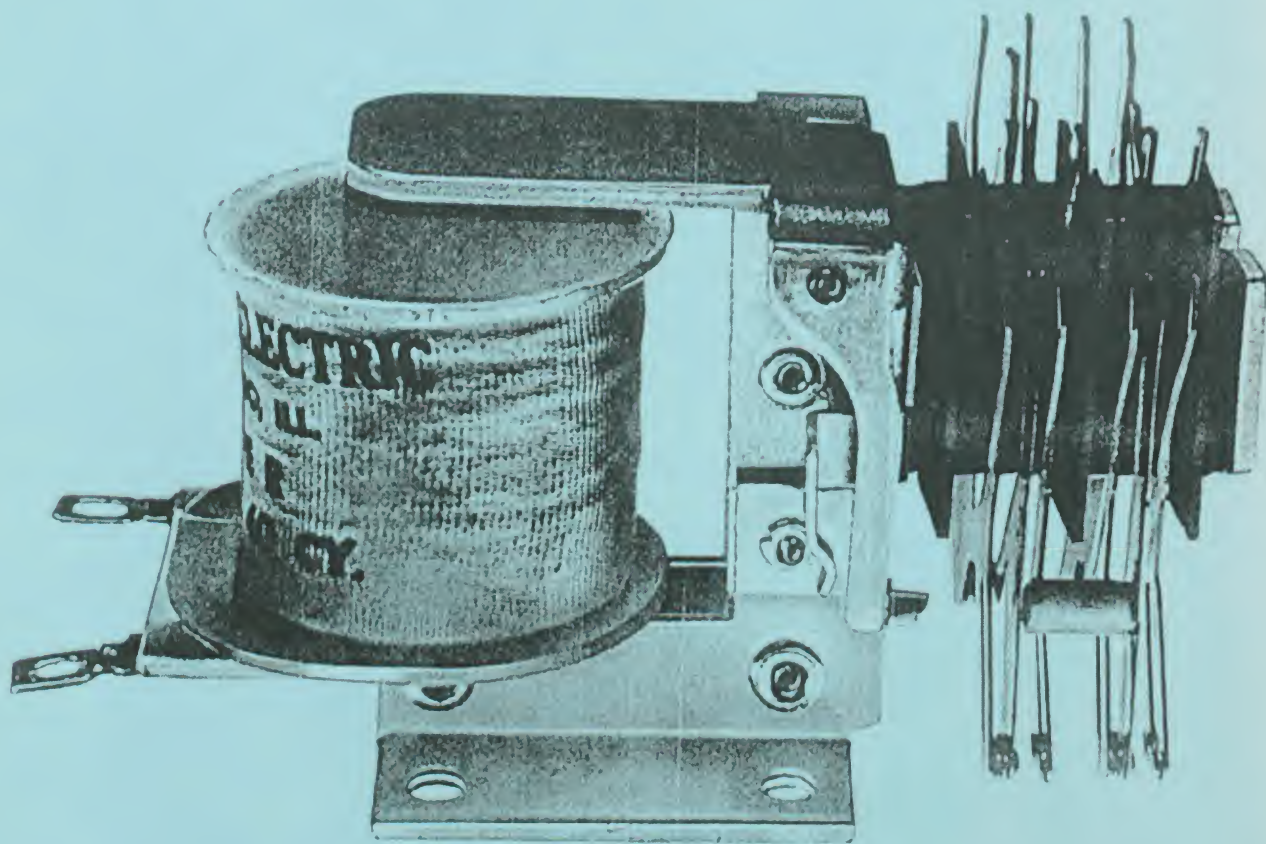


# PHYSICS OF TECHNOLOGY



## THE SOLENOID

ELECTRICITY AND MAGNETISM



# THE SOLENOID

A Module on Electricity and Magnetism

SUNY at Binghamton

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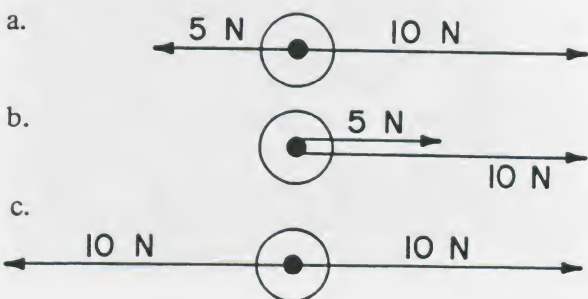


# The Solenoid

## PREREQUISITES

Before you start this module you will need a little basic knowledge of force, torque, and electric current. The questions below are a self-test, so you will know whether you are ready to start the module. If you can answer all of them, you should have no difficulty. If you have trouble with any of them, get some help from your teacher or another student. Then, you will be ready to proceed.

- Two forces act on a single object, as shown in the three cases below. What is the net force on the object in each case?



- Which of the following are correct units for electric current?

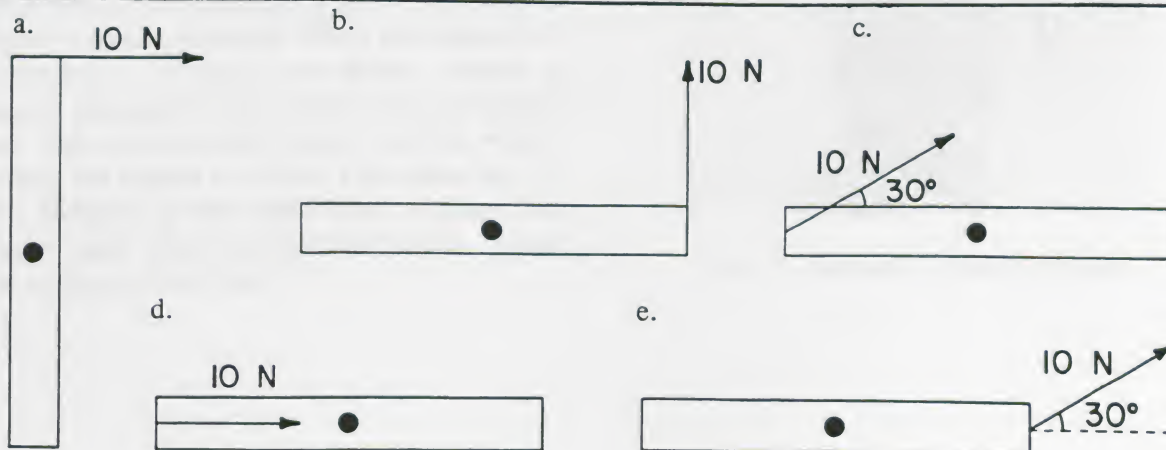
- Volts
- Amperes
- Ohms

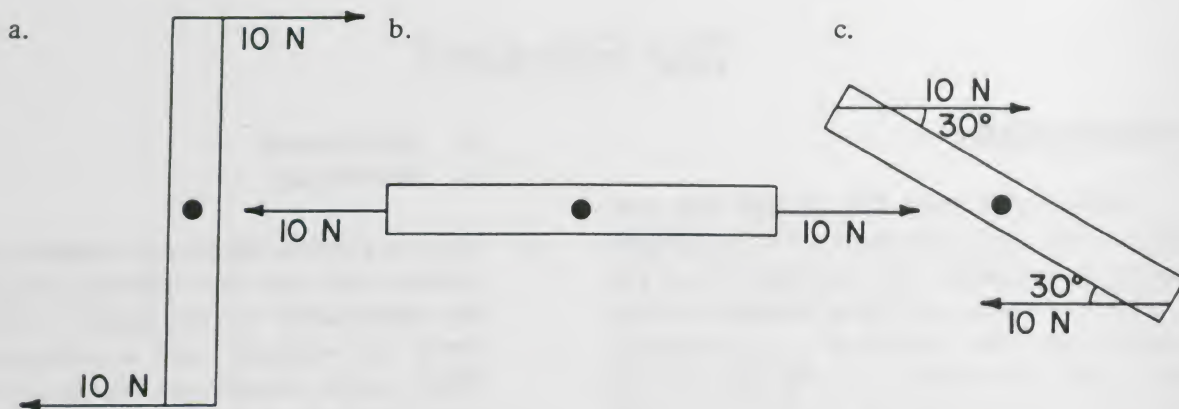
- Milliamperes
- Microfarads

- You are given a battery and a resistor. In addition you have the following measuring instruments: an ohmmeter, a voltmeter, an ammeter, and a wattmeter. Which meter would you use and how would you wire it to find the current the battery will produce in the resistor?
- Which of the following is a correct abbreviation for the units for torque, and what is it an abbreviation for?

- N
- Nm
- J
- W
- kg

- In each of the cases shown below, a force of 10 N is exerted on the end of a meter stick which has an axis at its center. What are the magnitude and sense (clockwise or counter-clockwise) of the torque on the meter stick in each case pictured below?
- A force-couple acts on a meter stick. Find the net force and the torque about the axis through the center of the meter stick in each case shown on page 2.



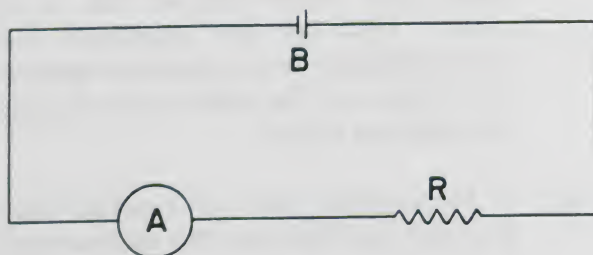


### Answers to Prerequisites Test

1. a. 5 N to the right  
b. 15 N to the right  
c. Zero

2. b and d

3. Use the ammeter and wire it in series with the battery and resistor, as shown in the circuit diagram.



4. b. Newton · meters. These same units, called joules, are used for energy and work. But torque is a very different physical quantity and the unit Nm cannot be called a joule when measuring torques.
5. a. 5 Nm, clockwise  
b. 5 Nm, counter-clockwise  
c. 2.5 Nm, clockwise  
d. Zero  
e. 2.5 Nm, counter-clockwise
6. In all three cases, the net force is zero.
  - a. 10 Nm, clockwise
  - b. Zero
  - c. 5 Nm, clockwise



## INTRODUCTION

The solenoid is a device which uses electrical energy to produce mechanical motion. Solenoids are used for "remote control" of otherwise inaccessible parts of a system. In many applications, a solenoid is used not only in remote control but also for automatic control.

For example, pressing the button of a door chime closes a switch. The current in the circuit then activates a solenoid which strikes the chime bar and causes it to ring.

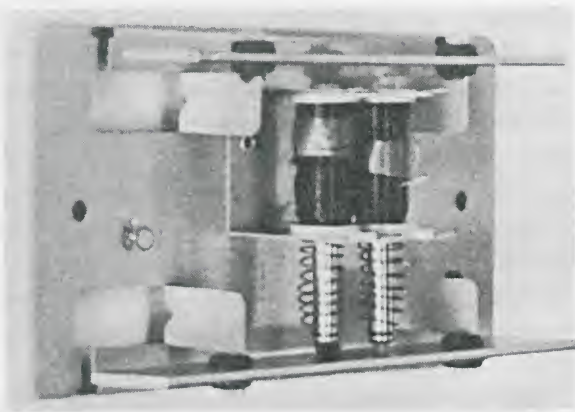


Figure 1. A doorbell mechanism, including two solenoids.

A train, passing over a certain portion of track, automatically activates a solenoid which in turn switches tracks and operates signals farther down the line.

Turning the ignition key in a car turns on a switch and energizes a solenoid. Figure 2 shows a starter solenoid. When the solenoid is energized, a plunger, not shown, pushes a copper disc against two copper contacts in the cap. This contact turns on the current, which causes the starter to crank. The other end of the plunger, at the same time, engages the starter gear with the flywheel, which causes the engine to turn over.

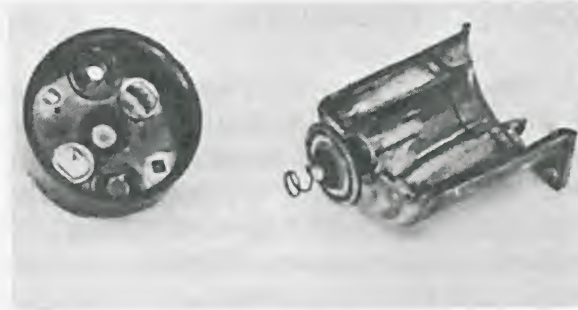


Figure 2. A starter solenoid.

In a washing machine, the water input to the tub is controlled by timer and temperature switches which activate hot and cold water solenoid-controlled valves. These valves control both the amount of water and the water temperature. The process is automatic since the valves are controlled either by a *sensing element*, such as a water-level switch, or by timing and temperature switches.

Solenoids have a large range of applications. How does this remarkable device work? What basic physical principles are involved? The remainder of this module provides some answers to these questions.

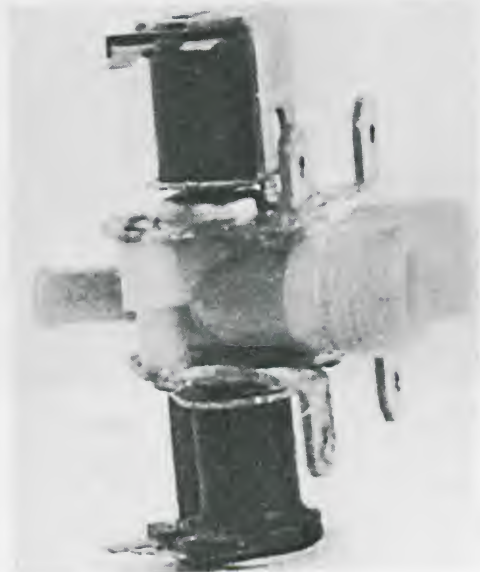


Figure 3. Automatic washer inlet valves.

## GOALS OF THIS MODULE

Students often wonder why a particular topic is being studied and what is supposed to be learned from it. Experience has shown that, when the goals of a course are made clear, it is easier for both the students and the teacher to achieve them. Therefore we list here, in the form of questions, the goals of this module. These are the most important points that we will try to cover.

1. How does a solenoid work? What principles of magnetism are essential to understanding the solenoid?
2. What is a magnetic field? What produces magnetic fields, what do they "look

like," and how can we describe them? In particular, what are the properties of magnetic fields produced by electric currents in straight wires, in loops, and in coils?

3. How do we predict the forces on magnets and currents when they are placed in the magnetic fields of other magnets and/or currents?
4. What is the origin of the magnetic forces which act on pieces of unmagnetized iron when they are placed in magnetic fields?
5. How can we use the observations made in this module to predict how the solenoid behaves in use in "real life"?



## SECTION A

### EXPERIMENT A-1. Study of a Solenoid

Although there are many different types and sizes of solenoids, the experiments in this module use a solenoid like the one shown in Figure 4. Look it over before connecting it to the power supply. The *plunger*, which moves along the axis (or center line) of the solenoid, is an iron cylinder. Can you guess how the plunger might behave when the solenoid is “energized”?

With the power turned off and the voltage turned to zero, connect the DC terminals of the power supply to the solenoid. (If your power supply does not have a built-in ammeter, you will need to put one in the circuit. Consult your instructor.) Set the voltage output to the 0- to 15-volt scale. Turn on the power supply and increase the voltage until you begin to feel a force on the plunger. In what direction does the force pull the plunger? Hold the plunger out and turn off the power supply. Then turn the power

supply on. Do you feel a force when the power is off?

Increase the current and read its value on the ammeter. Does the force on the plunger increase as the current increases? (The maximum allowable current for your solenoid is specified on the solenoid. Be careful that the current in the circuit does not exceed this maximum. If it does, the solenoid will overheat and be ruined.)

The solenoid requires an electric current to be energized. A larger current produces a larger force on the plunger. What causes the force on the plunger? In this module you will learn what causes the force and you will measure how the force depends on the current and the length of the plunger inside the core of the solenoid. As you may suspect, the operation of the solenoid depends on the phenomenon called *magnetism*.

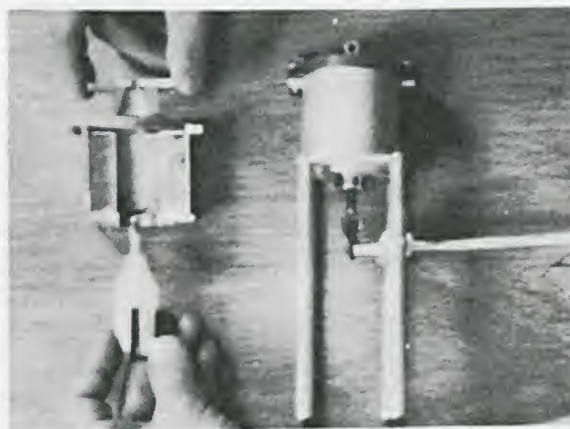
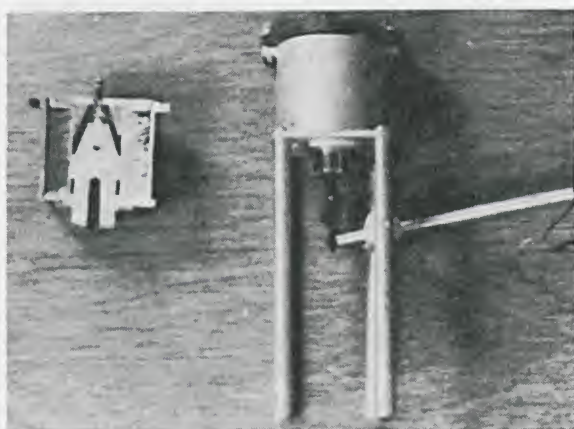


Figure 4.



## MAGNETISM

At some time or another, almost everyone has played with a magnet. You may have observed that a magnet attracts some objects, such as paper clips, and does not attract other objects, such as toothpicks. Materials which the magnet attracts are called *ferromagnetic* materials. Iron, cobalt, and a few alloys are the most common ferromagnetic materials. You may also know that two seemingly unrelated kinds of magnets exist. *Permanent* magnets are pieces of ferromagnetic material that are permanently magnetized. *Electromagnets* derive their magnetism from electric currents.

Other common magnets, called *ferrites*, are made of ferromagnetic compounds, especially ferric oxide. These magnets appear to behave differently from the magnets studied in this module.

You perhaps have played with two magnets and found that by turning one of the magnets around, the force which pulls the magnets together (the *attractive* force) is replaced by an equally strong force which pushes the magnets apart (a *repulsive* force). It is puzzling that one cannot see or feel any real connection between the magnets, yet the force is very real.

Saying that these properties are the result of something called "magnetism" does not provide any understanding of what magnetism is, or of why magnets behave as they do.

Hundreds of years ago scientists invented names to describe the behavior of magnets. They knew that the two ends of a magnet (call it magnet number 1) behave differently when brought near the same end of another magnet (call it number 2). As almost everyone

has seen, one end is attracted to magnet 2, and the other is repelled. But if magnet 2 is turned around, then the first magnet behaves exactly oppositely from the way it acted before. The end that previously was attracted is now repelled, and vice versa.

In order to describe such experiments, scientists labelled the ends of the magnets. They could have called them the "A" and "B" ends, or "+" and "-" ends, or anything else. But thousands of years earlier, people had found that a small magnet floating on a cork in water always tended to align itself with one end pointing northward and the other southward. (This was the first form of the magnetic compass.) Therefore, they named the ends, or *poles*, of the magnets the "north-pointing" and "south-pointing" poles. These names have been shortened to *north* and *south poles*. The behavior of two magnets may be described in terms of their *polarities* by saying that "like" poles repel and "unlike" poles attract, as shown in Figure 5.

One can do other experiments also. If you break a magnet into parts, you find that poles of opposite polarity always appear on either side of the break, as shown in Figure 6. The poles created by the break are always opposite in polarity and equal in strength, no matter how many times the magnet is broken. The new poles which appear are such that every piece of the magnet always has its own north and south poles. Scientists have been searching for hundreds of years for a single, isolated north or south magnetic pole—a *magnetic monopole* (monopole means "one pole"). None have yet been found. We conclude that magnetic poles always occur in pairs, as *magnetic dipoles* (dipole means "two poles"), with one north pole and one south pole, both of the same strength.

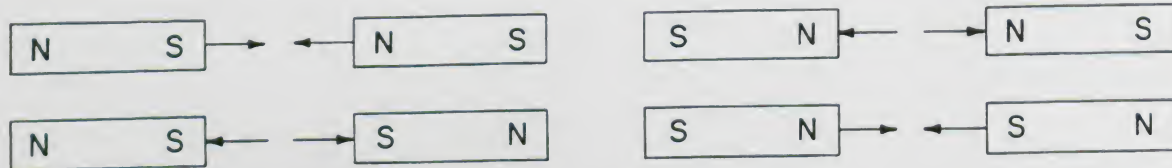


Figure 5. The attraction and repulsion of magnet poles. The arrows indicate the directions of the forces acting on the magnets.



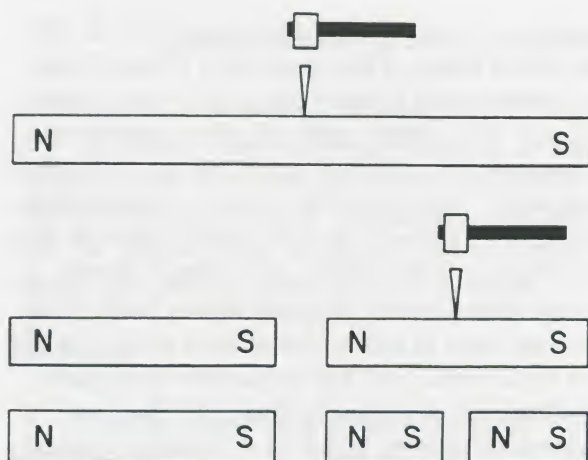


Figure 6.

In Experiment A-2, you will explore some of these things.

### EXPERIMENT A-2. Forces between Magnets

In this experiment, you will identify the poles of an unmarked magnet, using a compass. In addition, you will investigate the rules of polar attraction and repulsion, and observe what happens when you break a magnet.

#### Procedure

1. Identify the end of the compass needle which points north. This is the north pole of the compass. Actually, this pole may not point north because of the interaction of the compass magnet with structural steel in the building and/or large electric currents in the building's wiring.
2. Use the compass to identify the poles of the unmarked magnet, then label them.
3. Check your results by using the pre-labelled magnet.
4. Identify and label the poles on each end of the special, brittle magnet. Move the compass along the side of the magnet and look for evidence of poles in the body of the magnet. Break the magnet

and observe where the poles now occur and what their polarities are.

#### Questions

1. How do you decide which direction is geographic north?
2. We say that one end of a compass needle is a north-seeking magnetic pole. What is the polarity of the earth's magnetic pole that is located near the geographic north pole?

### THE MAGNETIC FIELD

The force exerted by a magnet on a piece of iron or another magnet is "felt" at some distance from the magnet, yet there is no apparent physical connection. You might wonder how the piece of iron, or the second magnet, "knows" that the first magnet is present.

One way to understand this phenomenon is to think of a magnet as somehow altering the space around it. This alteration, caused by the presence of the magnet, is called a magnetic "field of influence" or *field* for short. Any magnet, or piece of iron, placed in a magnetic field, interacts with that field and experiences a force. We can think of the field as the mechanism by which the strength and orientation of a magnet are communicated to other magnets, and to other materials.

In order to measure and describe the effects of magnets, we need more than just the idea that a field exists around a magnet. We need to be able to use numbers and directions to describe precisely the nature of the magnetic field at any point in space. Once the field at a point is known, we will be able to calculate the force that a magnetic pole at that point will experience. Then, we will have described completely the effects of the magnet. Thus, the magnetic field, although invisible, is a useful tool for analysis.

The first task is to define the magnetic field in exact terms. As for any quantity, we define it in terms of measurements that we

can make. Even though magnetic monopoles do not exist, we can approximate one by considering a pole which is far away from its opposite pole. The pole at either end of a long, thin magnet may be treated as a single magnetic pole. We will designate the (unknown) strength of such a pole by  $M$ . This strength can be defined in terms of the interaction between the magnetic pole and a magnetic pole whose strength is known.

Place a magnetic pole at the point in space where you want to measure the magnetic field. If a magnetic field exists at that point, the magnetic pole experiences a force which is measurable. The *magnetic field*  $B$  is defined as the force exerted on a north magnetic pole divided by the strength of the magnetic pole.

To “map” the entire magnetic field of a magnet, we could measure the force on a single magnetic north pole (called a *test pole*) at all points near the magnet, recording both the size and direction of the force at each point. Each of these forces could then be divided by the strength of the test pole to give the size of the magnetic field at each point. The *direction* of the field is defined to be the same as that of the force on a north pole. This procedure would give a size and direction for the magnetic field at every point in space. However, in practice there are just too many

points in the space around a magnet to do this for all of them. Field maps like Figure 7 may be constructed by placing a test pole at point a near the north pole of the magnet and measuring the force exerted on it. Then the magnetic field  $B$  at the point is calculated, and a small arrow is drawn which points in the direction of the field, and whose length is proportional to the strength of the field. Then the test pole is moved to point b at the tip of the first arrow, and the procedure is repeated, resulting in a second arrow. In this way, a pattern of arrows which is a “picture” of the magnetic field, is generated. We can make a neater and more useful diagram by drawing a smooth line to which each of these arrows is tangent. Such lines, called *magnetic field lines*, are another way of picturing the magnetic field. Some magnetic field lines for a bar magnet are shown in Figure 8. Each arrow points in the direction of the magnetic field at that location, and its length is proportional to the strength of the magnetic field at that point.

This is how magnetic fields are defined and measured “in theory.” But is this a practical way to map a magnetic field? Not really, since measuring the force on a magnetic pole at a large number of points in space is not as easy as we have made it sound. In addition, suitably isolated magnetic poles are difficult, if not impossible, to achieve.

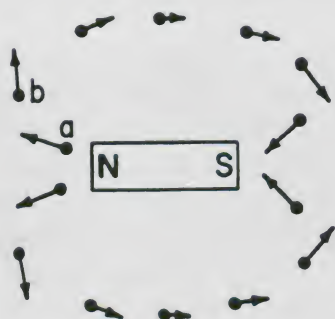


Figure 7.

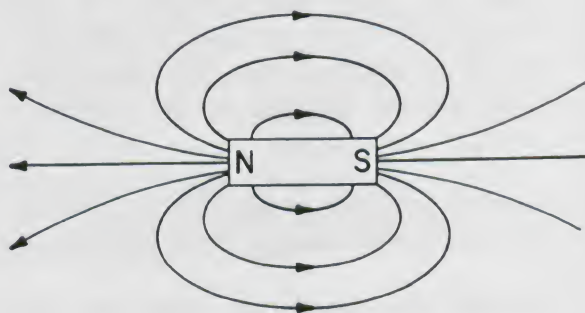


Figure 8.



### EXPERIMENT A-3. The Field of a Bar Magnet and Magnetic Induction

In this experiment, you will use a compass to “map” the magnetic field of a bar magnet. Then you can verify the field geometry by using iron filings. In addition, you will have an opportunity to study how a magnet attracts an unmagnetized piece of material.

#### Procedure

Plot the field of the bar magnet in the following way:

1. Place the bar magnet on a large piece of paper. Carefully draw a line around the magnet so that you can relocate it easily. Label the poles on the paper.
2. Place the compass near the north pole of the magnet. When the needle comes to rest, make a dot on the paper to mark the direction in which the north pole of the compass needle is pointing. Move the compass until the south pole of the needle points to the mark that you just made (see Figure 9). When the needle is again at rest, make another dot to mark again the direction that the north pole points. Repeat this procedure until you reach the south pole of the magnet.
3. Draw a smooth curve (a magnetic field line) through all the points.
4. Repeat this procedure. Draw a few field lines on each side of the magnet. Notice that each line begins at the north pole of the magnet and ends at the south pole. Do any of the field lines cross?
5. The rate at which the needle “quivers” in coming to a stop is proportional to the field strength. Try to estimate the relative field strengths in various regions of the field.
6. Now cover the magnet with the same piece of paper, being sure to place the magnet in the same location. Sprinkle some iron filings on the paper while gently tapping it. Be careful not to get any filings on the magnet. Do you see any relation between the pattern made by the filings and the field lines you just constructed?

#### The Effects of Magnets On Non-Magnetized Iron

1. Position a compass near the end of a nail or else a piece of *soft*\* iron. Does the

\*In the magnetic sense, “soft” is more easily magnetized than “hard” iron. The term does not refer to mechanical hardness. Soft iron does not feel soft to the touch.



Figure 9.

iron affect the compass needle? How does the compass needle behave when placed at the center of the nail (iron)? Can you devise a test to show that the nail (iron) is *not* magnetized?

2. Leave the compass near one end of the iron and bring a magnet close to the other end without letting it touch, as shown in Figure 10. Can you tell from

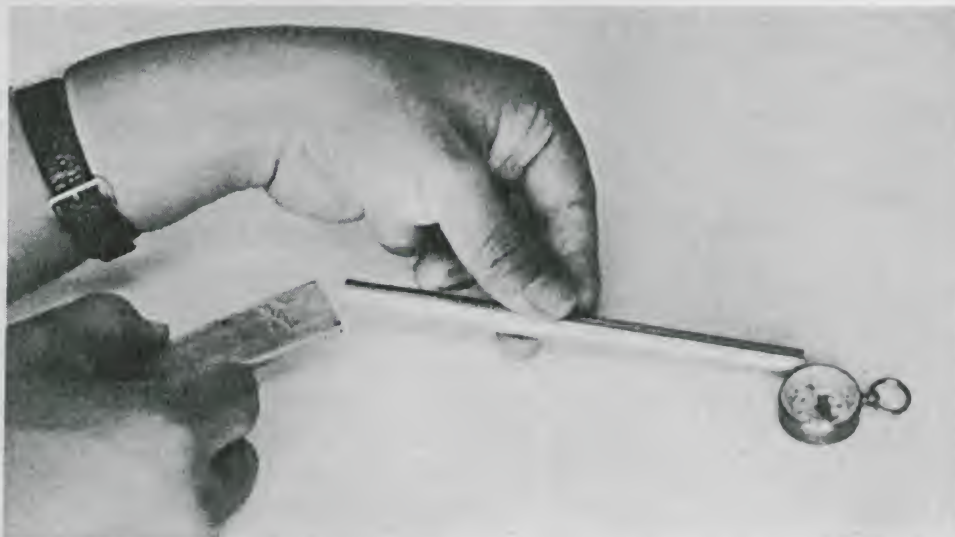


Figure 10.

the rate at which the needle vibrates if the strength of the magnetic field changes? What happens to the compass as you slowly pull the magnet away from the unmagnetized iron?

3. Reverse the magnet so that its opposite pole is near one end of the soft iron. How does this affect the compass? Does the "polarity" of the soft iron change?

### Questions

1. What does a magnetic field arrow signify and in what direction does it point?
2. Why do the iron filings align themselves along the magnetic field lines?
3. Where is the strongest region of the field? What happens to the magnetic

field strength as you move farther away from a magnet?

4. Without doing more mapping, can you give the direction of the magnetic field at a point which is in between two of the field lines on your plot? Explain.
5. Give two possible reasons for the deflection of the compass by the piece of soft

iron before you brought the magnet near it.

6. Explain how the chain of pins in Figure 11 is formed.

In the laboratory, a simple magnetic compass is used to plot magnetic fields. The compass is a very light magnet placed on a low-friction pivot so that it rotates easily. Figure 12 shows the forces acting on a compass needle in a magnetic field. The long arrows are portions of magnetic field lines, and the arrow at the end of each pole shows the direction of the force acting on that pole. When the compass is oriented as shown in Figure 12A, the forces on both the north and south poles tend to rotate the compass until it is parallel to the field line. If the compass needle overshoots the parallel position, as shown in Figure 12B, the forces





Figure 11.

act to reverse the rotation and turn the needle back toward the parallel position. The needle quivers, or *oscillates*, as it settles down. When it does come to rest, the needle is parallel to the field and the north pole points in the direction of the magnetic field, as shown in Figure 12C.

As you did Experiment A-3, you observed the needle swing back and forth on either side of the field line as it reached its final direction, much as a tight cable vibrates when displaced from its stable position. Just as a tighter cable vibrates faster than a loose one, in a strong magnetic field the compass vibrates faster than in a weak field. The rate

at which the needle vibrates can be used as a measure of relative field strengths in the experiments.

The fact that a compass aligns itself with the magnetic field is, of course, what makes it valuable when you are in the woods. It aligns itself with the earth's magnetic field and points in a generally northward direction. However, you must worry about its reliability if you are carrying a rifle or knife while reading a compass.

When an unmagnetized piece of iron is brought near a magnet, it acquires properties similar to those of the magnet. The magnetic field set up by the magnet rearranges the internal structure of the unmagnetized piece of iron in such a way that it becomes *magnetized*. In particular, the iron is magnetized in a way that results in a force of attraction between the magnet and the piece of iron. The magnetization process is called *magnetic induction*, and we say that magnetic poles are *induced* in the iron. Both a north and a south pole are induced in the iron. The induced pole that is nearest to a pole of the magnet has a polarity opposite to that of the inducing pole (see Figure 13). Since unlike poles attract, the piece of iron is attracted to the permanent magnet. If another piece of iron is now introduced, the same process takes place, and it too is attracted. This property explains the behavior of the iron filings you used in Experiment A-3. When iron filings are placed in a magnetic field, poles are induced in each filing, and each one lines up along the magnetic field lines, just as a small compass needle does.

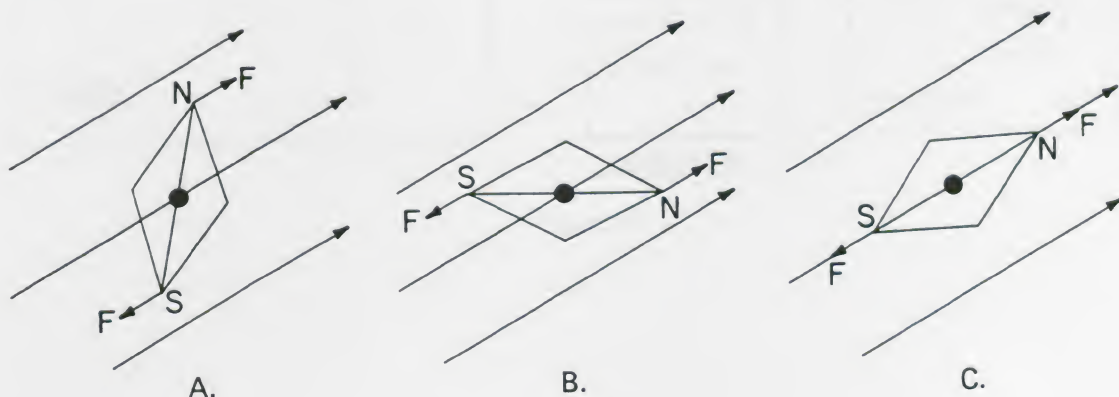


Figure 12.



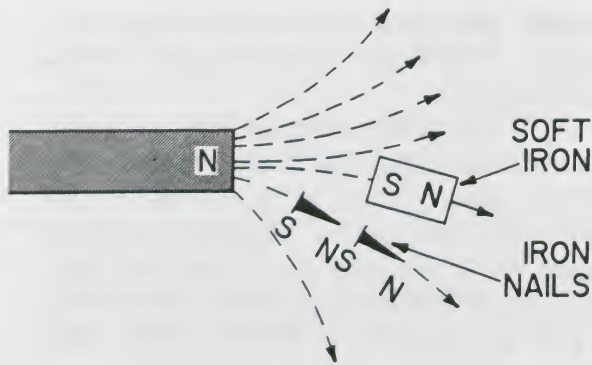


Figure 13.

### The Solenoid

The name of the solenoid refers to the “coil” shape in which the wire is wound. Any wire which is wound in a helical shape, like the threads of a bolt, is said to have a solenoidal shape and is called a solenoid. However, in this module we shall refer to the device consisting of the wire coil and plunger as a solenoid. We will refer to the wire as the *solenoidal winding*, or *winding*.

The principle of magnetic induction underlies the operation of the solenoid. The plunger of a solenoid is an unmagnetized piece of iron, and the solenoidal winding is a magnet which can be turned on and off. An electric current in the solenoidal winding turns it into an electromagnet which induces magnetic poles in the plunger. Thus the plunger is pulled toward the solenoidal winding.

### MAGNETIC FIELD OF A CURRENT

In order to explain the operation of the solenoid by assuming that the solenoidal winding is an electromagnet, which can be turned on and off, one must study the magnetic effects of electric currents. In Experiments A-4 and A-5, you will examine the magnetic field produced by a current in a straight wire, the field of a current loop, and the field produced by current in a solenoidal winding.

## EXPERIMENT A-4. The Magnetic Field of a Current in a Straight Wire

In this experiment, you will use a compass to map the magnetic field produced by a current in a wire. As before, you will verify the geometry of the field by using iron filings.

### Procedure

**CAUTION:** The circuits which you will encounter in this module are **dangerous** when energized! Do not touch exposed wires or terminals! Serious injury may result.

1. Set up the apparatus as in Figure 14.
2. Adjust the power supply so that a current of several amperes flows in the circuit. If the wires get hot and smoke, reduce the current.
3. Use the compass to map the magnetic field near the wire by taking point-to-point direction markings as you did for the bar magnet.
4. Plot a few field lines and indicate on each the direction of the magnetic field.

5. Verify the field-line geometry by sprinkling iron filings on the platform while tapping it gently.
6. Reverse the current in the wire. To do this, turn off the power supply, and reverse the leads. Then repeat steps 2-5.

### Questions

1. Sketch the magnetic field around a current-carrying wire, indicating the direction of the field and the direction of the current.
2. Does the magnetic field due to a current in a straight wire have north and south poles?
3. Can you devise a rule that will predict the direction of the field if you know the direction of the current?
4. Can you guess how this experiment is related to the basic operation of the solenoid?

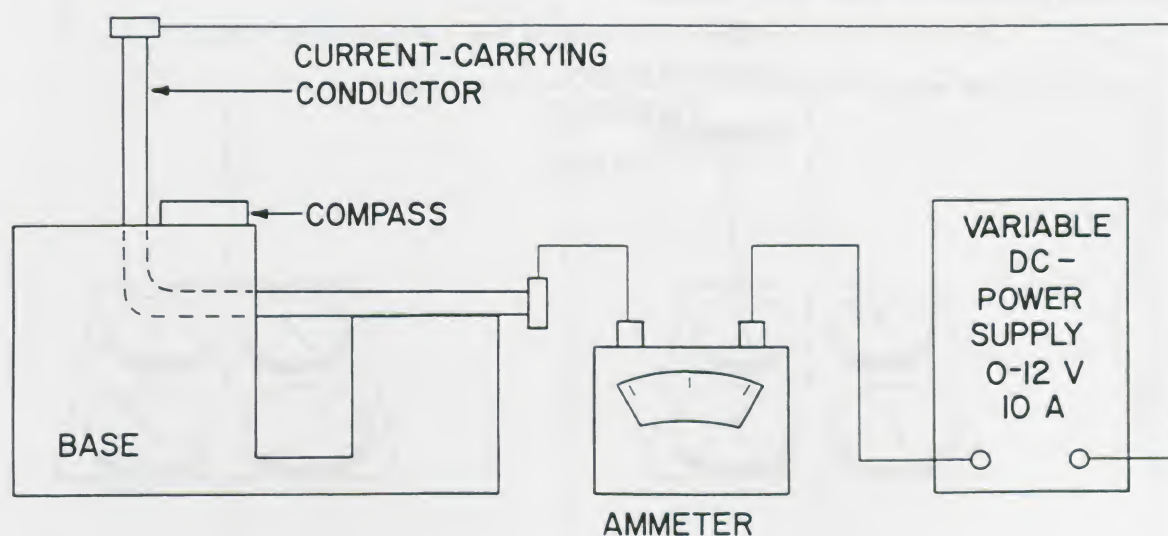


Figure 14.



## FIELD OF A STRAIGHT WIRE AND THE RIGHT-HAND RULE

An electric current in a conductor creates a magnetic field in the space around the conductor. This experimental fact is an indication of the existence of a basic and fundamental relationship between the seemingly unrelated phenomena of electricity and magnetism. Moving electric charges create a magnetic field. Although this module will not explore the implications of this, you should be aware that electricity and magnetism are two aspects of a more general phenomenon called electromagnetism. The simplest electric current path is a single straight wire, with the other parts of the circuit far enough away from it so that their effects are small. This was the situation in Experiment A-4, and Figure 15 shows the arrangement in *schematic* form. Before the power supply is connected, no current is present and the compasses point in the direction of the net magnetic field due to the earth and other sources, such as steel beams in the building. When the current is turned on, electric charge flows through the wire, and the compass needles are immediately affected. You found that the field pattern thus produced is made up of concen-

tric circles, centered on the wire, and lying in planes perpendicular to the wire. (See Figure 16.)

Were you able to devise a rule to predict the direction of the field if you know the direction of the current? If so, your rule may be a good way to remember how to relate the current and field directions. Ask your instructor about it if you are not sure whether or not your rule is reliable.

One such rule which is quite useful is called the "Right-Hand Rule": *If a wire carrying a current is held in the right hand with the thumb pointing in the direction of the current, the fingers curl in the direction of the magnetic field.* This rule is illustrated in Figure 17. The direction of the current is the direction in which positive charges will flow (from the positive terminal of the power supply through the circuit and back to the negative terminal).

The straight wire provides a magnetic field which can be turned on and off. The solenoid does the same thing but it uses a coil of wire rather than a straight wire. Therefore, in Experiment A-5, you will examine the magnetic field produced by a current loop, and then that of a solenoidal winding.

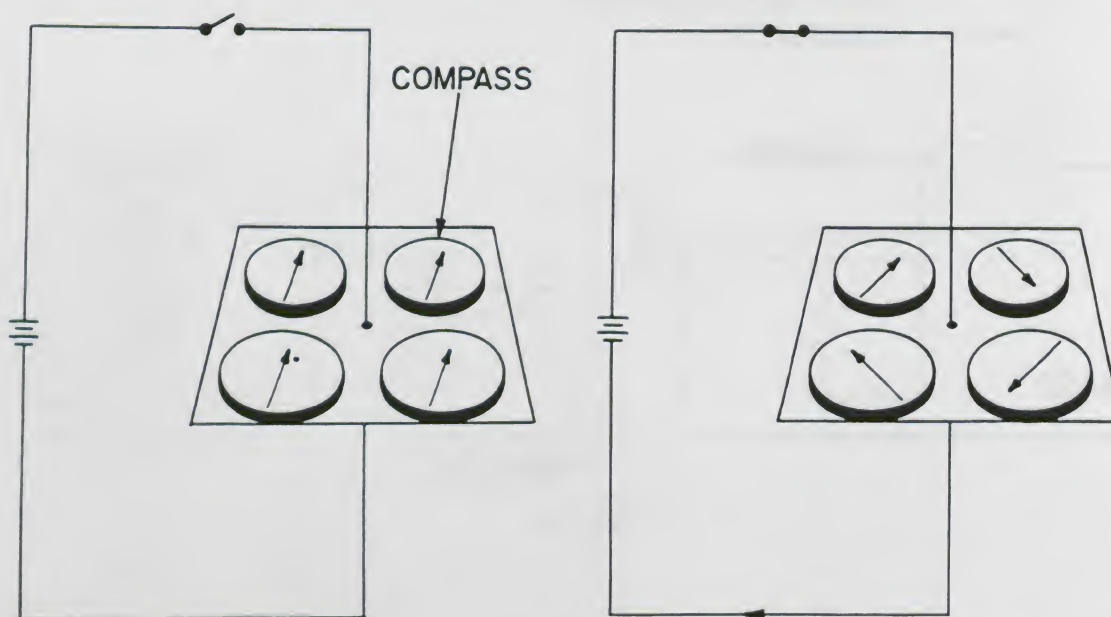


Figure 15.



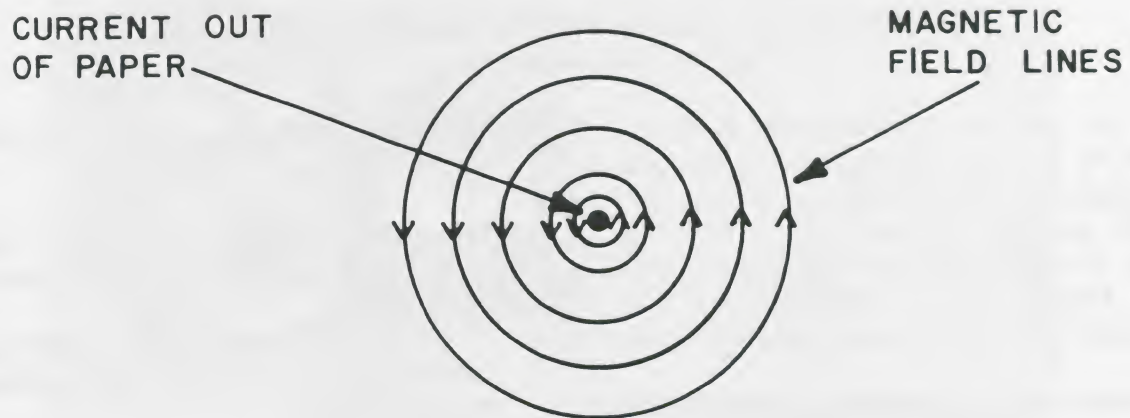


Figure 16.

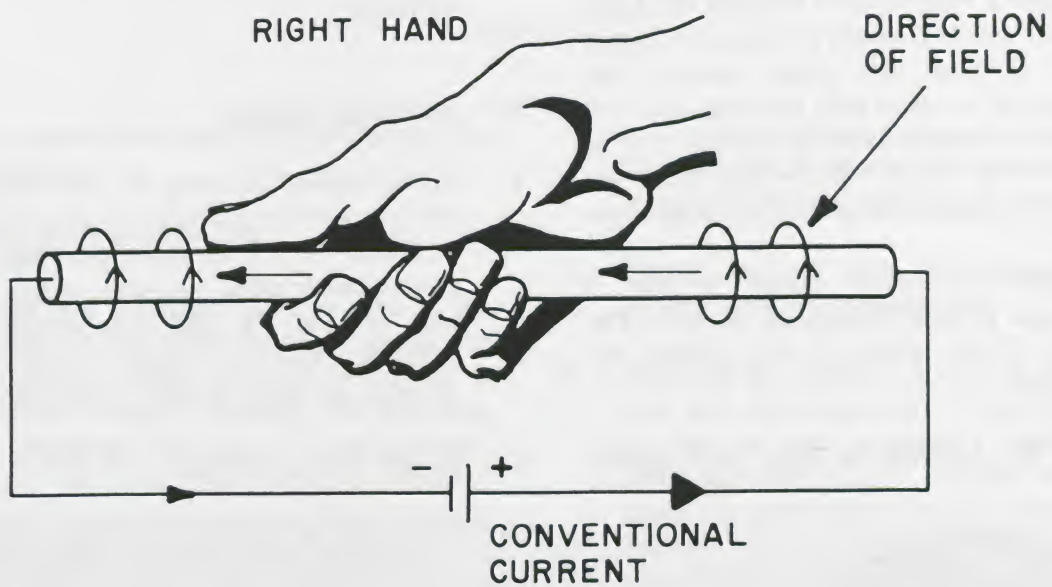


Figure 17.

## EXPERIMENT A-5. Magnetic Field Produced by a Current Loop and by a Solenoidal Winding

You will map the magnetic field produced by a current-carrying loop of wire, and the magnetic field of a current-carrying solenoidal winding. You will also observe the effect of placing an iron core into the solenoid. Finally, you will study the force the solenoid exerts on a magnet inserted into it.

### Procedure

#### A. Single-current loop

1. Thread a loop of wire through the holes in the plotting board as shown in Figure 18. Connect the power supply and ammeter in series with the loop. Cut two pieces of paper to fit around the loop as shown in Figure 18. **Do not touch the wires or terminals when the circuit is on.**
2. Adjust the power supply so that a current of several amperes flows in the loop. If the wires get hot, reduce the current.
3. Use the compass to map the magnetic

field as you have done in earlier experiments.

4. Plot several field lines and indicate the direction of the magnetic field for each.
5. Check the field geometry by sprinkling iron filings on the paper while tapping the paper gently.
6. Reverse the current in the loop and use the compass to see how the field is affected.

#### B. Solenoidal winding

1. Thread about 20 turns of wire through the holes in the plotting board as shown in Figure 19. Try to make each loop the same circular shape and size, so that you end up with a coil that outlines a cylinder. This is called a *solenoidal winding* or, more simply, a *solenoid*.
2. Repeat steps 2 through 6 of part A. Try

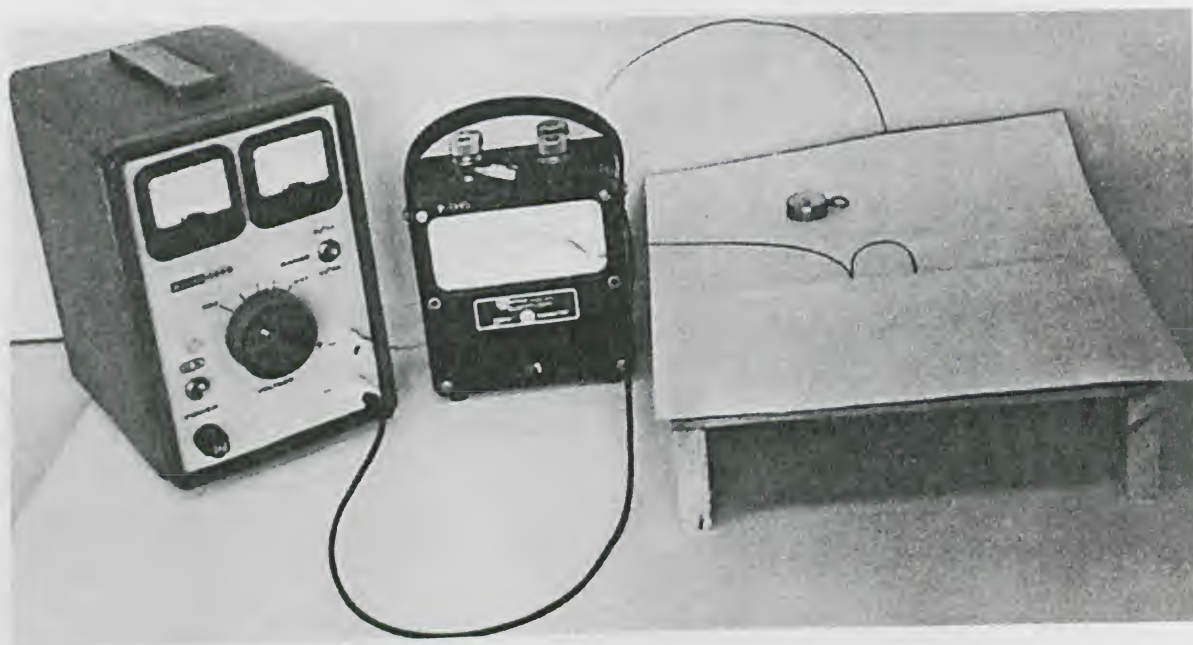


Figure 18. Mapping the magnetic field lines of a single loop of wire.



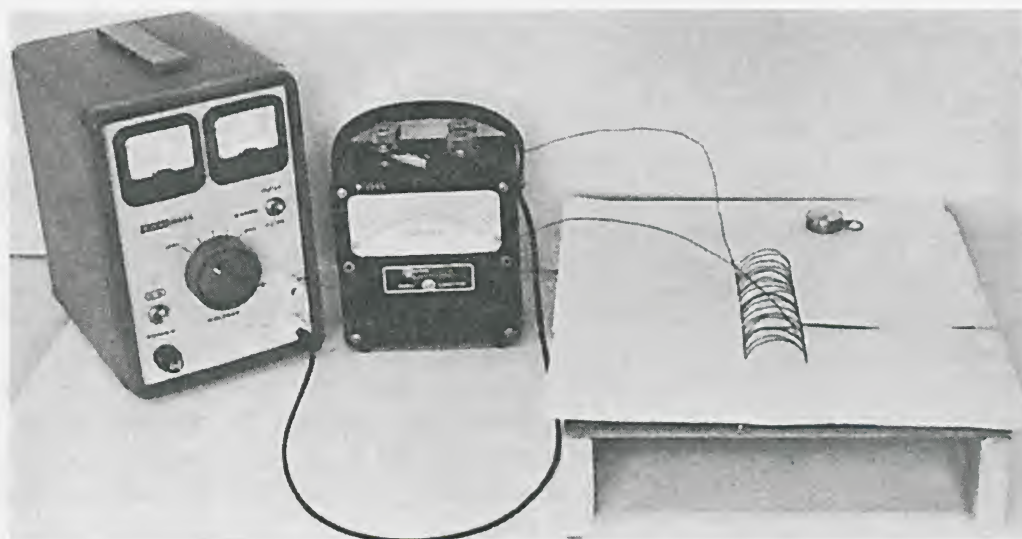


Figure 19.

to map at least one field line through the cylinder.

#### C. Effects of an iron core

1. Place the compass near the end of the solenoidal winding and shake gently. As it settles down, note how rapidly it vibrates. Without moving the compass, or changing the current in the solenoid, slide the iron core into the winding. Again shake the compass. Does it appear that the magnetic field strength has changed? Does the presence of the iron core increase or decrease the field strength?

#### D. Force exerted on a magnet

1. Turn the plotting board on its end and suspend a bar magnet from a spring balance so that about a third of the magnet is inserted into the solenoidal winding (see Figure 20). Record the weight of the magnet, as indicated by the spring balance.
2. Turn on a current of several amperes (you may need the maximum output of the power supply) and record the new spring balance reading. Reverse the

current connections and repeat. Diagram your results, indicating force directions, current directions, and magnet polarities.

3. Repeat step 2 with the magnet suspended from its opposite end.
4. Turn off the current and insert the iron core into the lower end of the solenoid so that its end is about a centimeter from the end of the hanging magnet. Using the same value of current as before and the current direction which causes an attractive force on the magnet, record



Figure 20.

the new spring-balance reading. Compare this force to the force produced when the iron core is not present.

### Questions

1. Show that the right-hand rule predicts the direction of the fields which you observed for the single-current loop and for the solenoidal winding.
2. Compare the general shape of the field of the bar magnet to that of the solenoid.
3. How do you suppose the maximum strength of the field of the solenoid compares to that of the single loop when the current in each is the same?
4. Does the presence of the iron core increase or decrease the strength of the magnetic field near the end of the solenoidal winding?
5. Sketch the magnetic field around the solenoid and the bar magnet for steps 2, 3, and 4 of part D. Explain the forces that you observed.

### Discussion of Experiment A-5

The magnetic field lines around a circular loop of wire which is carrying a current are just what you would expect if you bend a long, straight, current-carrying wire into a loop. The lines are still continuous curves around the wire, but they are no longer circles. They seem to be flattened somewhat as they pass through the center of the loop. The configuration of field lines is "doughnut-shaped."

Whenever the magnetic field (such as several current loops) at a point in space is due to more than one source, the total field at the point is just the sum of the fields at that point due to each of the field sources present. It does not matter whether the contributions to the total field are due to currents or permanent magnets, or combinations of both.

When adding the fields together, both the directions of the magnetic field contributions and the size of each contribution must be considered.

The solenoidal winding may be regarded as a stack of current loops. You probably observed that the field lines outside the winding are very similar to those of a bar magnet. The magnetic field lines inside the winding are parallel to the axis of the winding, and the field is relatively strong inside the winding. In Section B of this module, the fields of a current loop and a solenoidal winding will be analyzed mathematically.

Finally, you saw that the presence of the iron core in the solenoid increased the magnetic force exerted on the hanging magnet. From this observation, one can infer that the iron core increases the magnetic field strength of the solenoidal winding.

### SUMMARY

A solenoid is a helical coil of wire that produces a force on, and a subsequent motion of, a plunger. The force is produced by the magnetism associated with an electric current in a coil of wire.

Permanent magnets always attract unmagnetized pieces of iron and may exert either an attractive or a repulsive force on other permanent magnets.

The magnetic effects of permanent magnets appear to be concentrated near the ends. We call such regions magnetic poles.

Every magnet has two magnetic poles which behave differently; we call these two kinds north poles and south poles. North poles repel other north poles; south poles repel other south poles; north poles attract south poles and vice versa. Each pole is characterized by a property called pole strength.

A magnet alters the space around it to create a magnetic field. The magnitude of the magnetic field is defined to be equal to the force on a magnetic pole divided by the strength of the magnetic pole. The direction of the field is the direction of the force on a



north pole. Magnetic field lines can be drawn to indicate the direction of the field.

Permanent magnets induce magnetic poles in previously unmagnetized pieces of iron. Any induced pole is opposite in kind to the permanent pole responsible for inducing it.

Current-carrying wires create magnetic fields. The field lines near a long straight wire

are circles concentric with the wire and in planes perpendicular to the wire.

The magnetic field lines near a circular-current loop are closed curves that pass through the plane of the loop. When several current loops are lined up along an axis in what is called a solenoidal winding, the field lines inside the solenoid are straight lines parallel to the axis.

## SECTION B

### FORCES AND FIELDS OF PERMANENT MAGNETS

Since the solenoid is used to produce a mechanical force, we need to be able to predict and measure the force applied to the plunger. This force is the force of attraction between the solenoidal winding and the plunger, which is like the induced magnetic force you saw in the last part of Experiment A-5. In order to study this force more carefully, we must introduce mathematical relationships among pole strength, field, and the forces between poles.

You have seen that a magnetic field exists around permanent magnets and current-carrying wires. You know the shapes of these fields and some of their properties. To understand the forces in the solenoid, you will also need to know how to find the numerical value of the magnetic field strength at points in the fields of currents and magnets. We will start with permanent magnets.

Early experiments on magnetism led to *Coulomb's Law* for the force between two magnetic poles, of strengths  $m$  and  $m'$  (Figure 21). The force between poles is directly proportional to the product of the pole strengths and inversely proportional to the square of the distance between them. Algebraically, this law is stated as follows:

$$F = k \frac{mm'}{r^2} \quad (1)$$

The symbols and their units in the SI (Standard International or, more properly, *Système Internationale*) system of units are:  $F$  = force of attraction or repulsion between two poles in newtons (N);  $m$  and  $m'$  = the two pole strengths in ampere-meters ( $A \cdot m$ );  $r$  = distance between the poles in meters (m);  $k$  = a constant which is equal to  $10^{-7}$  newtons/(ampere)<sup>2</sup> ( $N/A^2$ ). Thus, in SI units, Coulomb's Law becomes

$$F = (10^{-7} \text{ N/A}^2) \frac{mm'}{r^2}$$

The magnets in Figure 21 are long and narrow, so that the effects observed are mostly due to the two closest poles. If so, then we can ignore the much weaker effects of the forces arising from the poles at the other ends of the magnets. If magnetic monopoles existed, we would use them instead of these long thin magnets. Also if magnetic monopoles existed, Equation (1) would be precise. In the absence of monopoles, it is only approximate.

In Experiment B-1, you will measure the forces between magnetic poles.

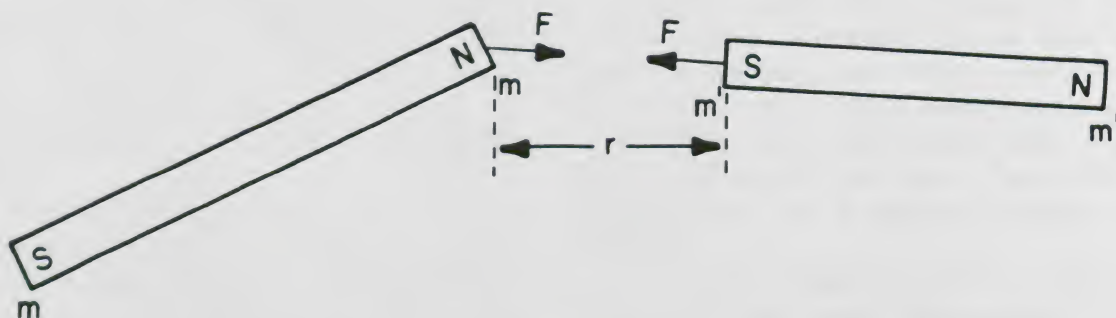


Figure 21.



## EXPERIMENT B-1. Force between Magnetic Poles

### Procedure

1. Set up the apparatus as shown in Figure 22.
2. Place a *shim* (ordinary bond paper anywhere from 0.1-mm to 0.25-mm thick) between the magnets. Measure the thickness of the shim with a micrometer and record it. The shim provides a known separation between the two magnets.
3. Gently pull magnet #1 until the magnets separate, recording the scale reading just before separation. Repeat this three times and average the three readings.
4. Repeat step 3 using a thicker shim. Any increase is acceptable as long as you can still measure the force.
5. Now you could calculate the product of the pole strengths of the magnets by

using Coulomb's Law. However, at this point, there is no way to find  $m$  and  $m'$  separately without further information. For example, the force each exerts on an unmagnetized piece of iron in identical conditions could be measured. For the present purposes, it will be adequate to assume that the two poles are identical. Then, for  $m' = m$ :

$$F = k \frac{m^2}{r^2} \quad (2)$$

However, one should not assume that  $r$  in the equation is equal to the shim thickness. If you draw many lines of force, they seem to originate approximately from a small region inside the end of the magnet. Thus, it is a more reasonable approximation to assume that each magnetic pole is at a point located some distance  $d_0$  from the end of the magnet as shown in Figure 23. Then,  $r =$

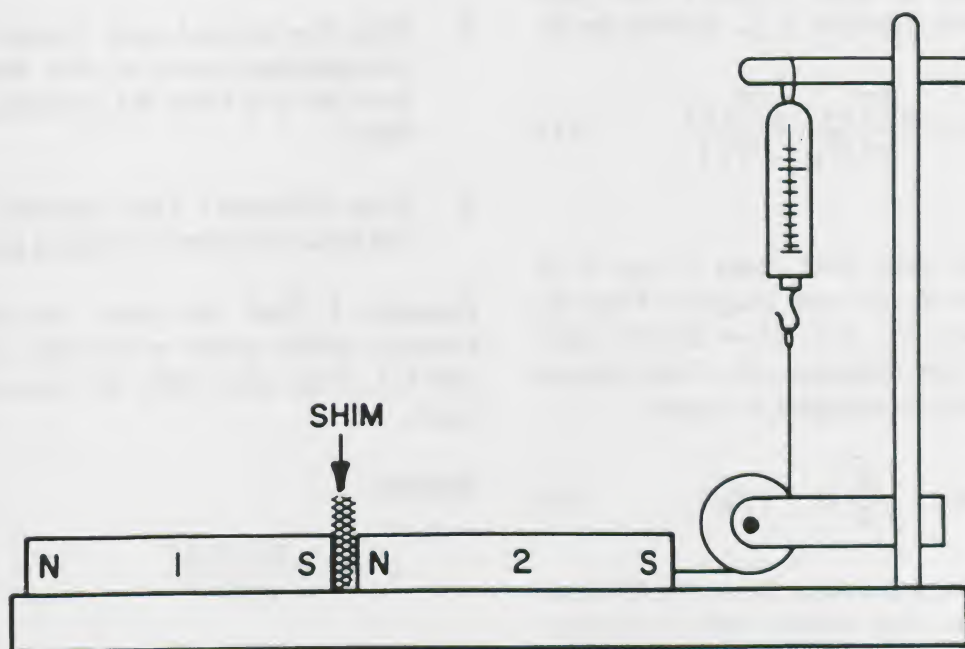


Figure 22.

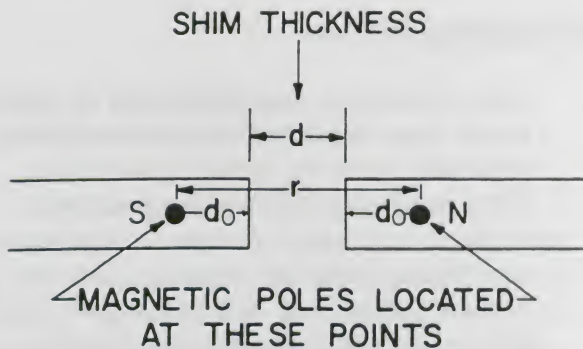


Figure 23.

$2d_0 + d$ , where  $d$  is the shim thickness, and Equation (2) becomes

$$m^2 k = F(d + 2d_0)^2 \quad (3)$$

The quantity on the left does not change in this experiment. You can therefore equate the right sides, as evaluated in steps 3 and 4. That is, you can write

$$F_3(d_3 + 2d_0)^2 = F_4(d_4 + 2d_0)^2$$

where the subscripts 3 and 4 refer to the values of force and shim thickness measured in steps 3 and 4. The only unknown quantity is  $d_0$ . Solving for  $d_0$  yields

$$d_0 = \frac{\sqrt{F_4} d_4 - \sqrt{F_3} d_3}{2(\sqrt{F_3} - \sqrt{F_4})} \quad (4)$$

Use the data from steps 3 and 4 to compute  $d_0$  for your magnets. When  $d_0$  is known, the value of  $m$  may be computed from Equation (3), if we assume that the pole strengths are equal.

$$m = \sqrt{\frac{F}{k}} (d + 2d_0) \quad (5)$$

6. Repeat step 3, using a shim of a different thickness than those used previously. Before making a measurement, use Coulomb's Law to calculate what the force should be. Check your answer by doing the measurement, as described in step 3.

7. Repeat step 6 for two other shim sizes.
8. Repeat step 3 using magnet #2 and an unmagnetized piece of steel or iron of the same size as magnet #1.
9. Calculate the strength of the induced pole in the iron assuming that the pole strength of magnet #2 has not changed since step 3 and that  $d_0$  is the same for the induced pole as it is for the permanent pole.
10. Repeat step 8 using a larger value of shim thickness  $d$ .
11. Repeat step 9 for the data taken in step 10.

### Questions

1. When is the force greater—for a small shim thickness or a large one?
2. Is a graph of shim thickness versus magnetic force a curve or a straight line? Explain.
3. Does the induced pole strength of the unmagnetized piece of iron depend on how far it is from the permanent pole? Why?
4. Does Coulomb's Law correctly predict the forces measured in steps 6 and 7?

**Example 1.** Find the force of attraction between unlike poles of strength  $0.5 \text{ A}\cdot\text{m}$  and  $0.5 \text{ A}\cdot\text{m}$ , when they are placed 10 cm apart.

**Solution.**

$$\begin{aligned} F &= k \times \frac{0.5 \times 0.5}{(10^{-1})^2} \\ &= 10^{-7} \times \frac{.25}{10^{-2}} = 0.25 \times 10^{-5} \text{ N} \end{aligned}$$

**Problem 1.** If a magnetic south pole of



unknown strength is 0.2 m away from a pole of strength  $0.3 \text{ A}\cdot\text{m}$  and feels a force of  $6 \times 10^{-8} \text{ N}$ , what is the strength of the unknown pole?

**Problem 2.** Two long bar magnets are parallel to each other. A line perpendicular to both

magnets passes through both north poles and another line perpendicular to both magnets passes through both south poles. All four poles have strength equal to  $2.0 \text{ A}\cdot\text{m}$ . The force tending to move the magnets farther apart is  $2 \times 10^{-5} \text{ N}$ . What is the perpendicular distance between the magnets?

## MAGNETIC FIELD

We can now use Equation (1) to calculate the magnetic field in the region of a single magnetic pole. Remember that the magnetic field is defined as the force that a magnetic test pole feels, divided by the test pole strength:

$$B \equiv \frac{F}{m} \quad (6)$$

The units of magnetic field are newtons per ampere-meter ( $\text{N}/\text{A}\cdot\text{m}$ ). One  $\text{N}/(\text{A}\cdot\text{m})$  is called a tesla (T) in the SI system of units. Another common unit equal to 1 T is a weber per square meter ( $\text{Wb}/\text{m}^2$ ). A third common unit is the gauss (G).  $1 \text{ T} = 10^4 \text{ G}$ .

Suppose that the north pole of known strength  $m$  in Figure 21 is a test pole which we can move around in the field of the pole  $m'$ . If we want to measure the magnetic field of the pole  $m'$ , we can do as we discussed in Section A when we defined the field. That is, we can put the test pole  $m$  at various points in the field, measure the force on it, and calculate  $B$  from Equation (6). However, by using Coulomb's Law, we can find the magnetic field strength another way. We know that in the field of the pole  $m'$ , the force  $F$  is given by Equation (1). We can calculate  $B$  by combining Equation (6) and Equation (1) as follows:

$$B \equiv \frac{F}{m} = k \frac{m'}{r^2} \quad (7)$$

This equation gives the magnetic field at any distance  $r$  from an isolated magnetic monopole of strength  $m'$ . Since magnetic monopoles don't exist, this equation is approximately correct only in the region near pole  $m'$ , where the effect of the other pole is small. When the effect of the other pole is large enough that it can't be neglected, then we have to use Equation (7) for each of the poles to calculate the separate components of the field produced by each of the poles. These separate components of the field due to the

two poles can then be added, taking the directions into account, to find the total field, as in Problem 3.

**Example 2.** A bar magnet is 8.0 cm long between poles and has poles of strength  $0.4 \text{ A}\cdot\text{m}$ . At a distance of 0.5 cm from the north pole, as indicated in Figure 24, the north pole influences the magnetic field strongly, but the south pole is too far away to contribute much to the field. What is the magnitude of this field, and what is its direction?

**Solution.**

$$\begin{aligned} B &= \frac{km}{r^2} = \frac{10^{-7} \times 0.4}{(0.5 \times 10^{-2})^2} \\ &= 1.6 \times 10^{-3} \text{ T} \end{aligned}$$

The field is directed away from the north pole. (This field is 289 times as great as that due to the south pole at this same point.)

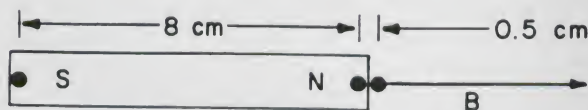


Figure 24.

**Problem 3.** At a distance 12 cm from the north pole of the magnet described in Example 2 and 20 cm from the south pole, as shown in Figure 25, the field is weaker than that calculated in Example 2. Partly this is because this point is farther away from the north pole, but also the contribution of the south pole to the field can no longer be neglected. What is the field at this point?

**Problem 4.** A bar magnet with pole strength  $0.2 \text{ A}\cdot\text{m}$  is placed in line with the bar magnet described in Example 2. It is 11.5 cm long. The south pole of the second magnet is put 0.5 cm from the north pole of the first magnet, and the north pole of the second magnet is then 12.0 cm from the north pole of the first magnet, as shown in Figure 26.



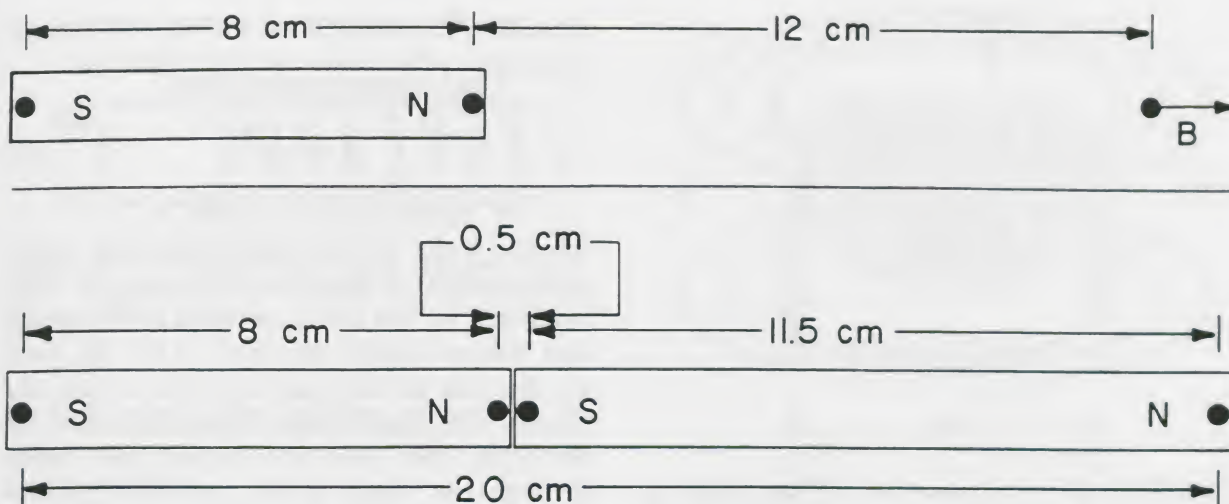


Figure 25 top. Figure 26 bottom.

What is the net force between the two magnets? Is it attractive or repulsive?

## THE FIELDS OF CURRENT-CARRYING WIRES

### Straight Wire

We have already studied the shape of the magnetic field induced around a current-carrying conductor. But we do not have an equation such as Equation (3) which would give the size of the magnetic field at various points for various currents. It is possible, of course, to discover such a law by conducting experiments. The results show that for a point a distance  $r$  away from a long straight wire carrying a current  $i$  (see Figure 27), the field is given by:

$$B = 2k \frac{i}{r} \quad (8)$$

where  $k$  is the same quantity we used in Coulomb's Law ( $k = 10^{-7} \text{ N/A}^2$ ),  $i$  is the current in amperes, and  $r$  is the perpendicular distance of the point in the field from the wire, measured in meters.

**Example 3.** Find the magnetic field strength ( $B$ ) at a point a perpendicular distance of 0.5 m from a straight wire carrying a current of 20 A.

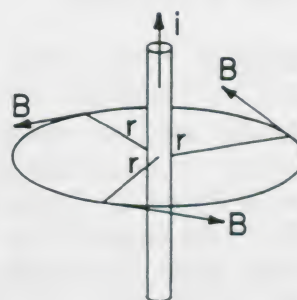


Figure 27.

**Solution.**

$$B = \frac{2 \times 10^{-7} \times 20}{0.5} = 8 \times 10^{-6} \text{ T}$$

**Example 4.** For the current in the above example, at what distance from the wire is the magnetic field equal to the earth's field? Assume the magnetic field of the earth to be  $B_e = 0.6 \text{ G}$ .

**Solution.** First find the earth's field in tesla.

$$0.6 \text{ G} \times 10^{-4} \frac{\text{T}}{\text{G}} = 0.6 \times 10^{-4} \text{ T}$$

Solve  $B = \frac{2k}{r}$  for  $r$ :

$$\begin{aligned}
 r &= \frac{2 ki}{B} \\
 &= \frac{2 \times 10^{-7} \times 20}{0.6 \times 10^{-4}} \\
 &= \frac{40 \times 10^{-7}}{0.6 \times 10^{-4}} \\
 &= 66.7 \times 10^{-3} \text{ m} \\
 &= 6.7 \text{ cm}
 \end{aligned}$$

Therefore at a distance of 6.7 cm from a conductor carrying a current of 20 A, the magnetic field due to the current is equal in magnitude to that of the earth.

### Single-Current Loop

The solenoid can be analyzed as a collection of single loops of current, such as the one shown in Figure 28. Try to apply the right-hand rule to the current loop. You will see that the field drawn agrees with the right-hand rule. No matter where you grasp the loop with your right hand, your fingers will be pointing through the loop in the same

direction. Experiment has shown, and calculations agree, that at the center of the loop

$$B = \frac{2 \pi ki}{R} \quad (9)$$

where  $i$  is the current through the loop and  $R$  is the radius of the loop. The magnetic field produced by the loop is stronger in the center and weaker outside the loop. Also, the field on the axis of the loop becomes weaker the further one moves away from the center of the loop. (The *axis* is a straight line drawn through the center of the loop and perpendicular to its plane. In other words, it has the same relation to the loop as an axle has to a wheel.)

### The Solenoidal Coil

We may now put together a whole series of current loops to form a solenoid, as in Figure 29. Experimentally it is found that the magnetic field does not change much from place to place inside the coil. At a particular point large contributions to the field due to some loops compensate for small contributions from others, and the resulting total

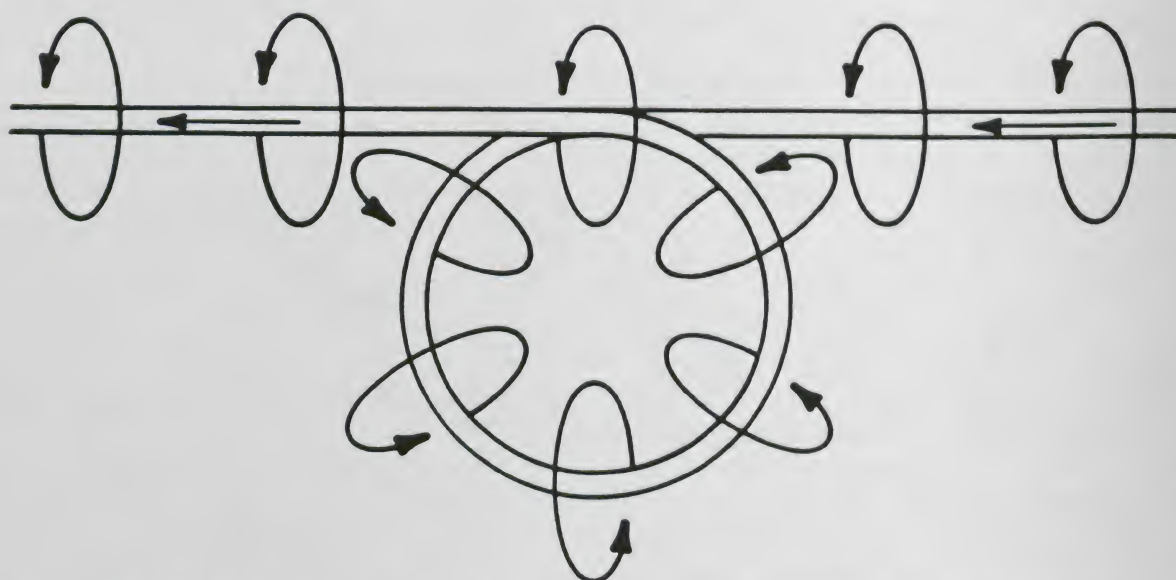


Figure 28. The field inside the loop is stronger than that near a straight wire carrying the same current because many magnetic field lines are added together to form the total field inside the loop.



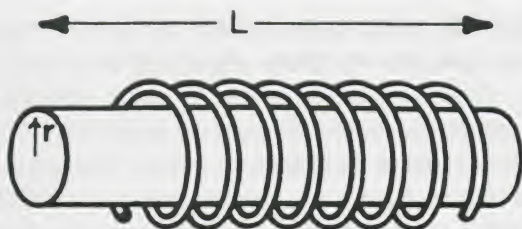


Figure 29. A solenoidal winding. It is shown as if it were wrapped around a paper tube, to make the illustration clearer.

field is remarkably uniform as one moves from one point to another inside the solenoidal winding.

What properties of the solenoid would the magnetic field depend on? The number of turns? The length? The radius? The current? The size of the field depends on some of these things, but the almost uniform field inside the winding is quite simple. Both measurement and calculation show that

$$B = 4\pi k \frac{Ni}{L} \quad (10)$$

everywhere inside a solenoidal winding (except near the ends), where  $N$  = number of turns,  $i$  = current in amperes, and  $L$  = length of solenoid. The field does *not* depend on the radius, as long as the solenoid's length is much larger than its radius.

The field inside the solenoid is stronger than that of a single loop, because the contributions from all of the current loops are in the same direction inside the solenoid and they all add up. You can see from Equation (10) that the field is stronger if there is a large number of turns. Try the right-hand rule on a few turns and convince yourself that the magnetic fields from all of the turns add together at any point near the winding. Notice that the field of Figure 30 and your plots of the field of a solenoidal winding done in Experiment A-5 look very much like the field plots of the bar magnet that you did in Experiment A-3.

**Example 5.** A solenoidal coil 10-cm long and

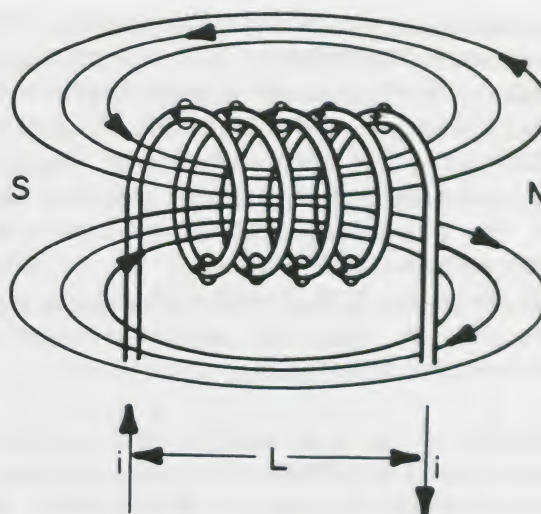


Figure 30. The magnetic field produced by a solenoid.

with 500 turns carries a current of 10 A. Calculate the magnetic field inside the solenoid.

**Solution.**

$$\begin{aligned} B &= \frac{4\pi \times 10^{-7} \times 5 \times 10^2 \times 10}{10^{-1}} \\ &= 2\pi \times 10^{-2} \text{ T} \\ &\cong 0.06 \text{ T} \end{aligned}$$

We now know how to produce a field that is a lot like the field of a bar magnet, but which can be turned on or off by throwing a switch that controls the current in the solenoidal winding.

Because some of the magnetic field does leak out between the turns, the magnetic field near the end of a long solenoid is one-half as large as in the interior, and is thus given by the expression

$$B = 2\pi k \frac{Ni}{L} \quad (11)$$

This expression gives the magnetic field available to magnetize the solenoid plunger when it is just outside the windings. The



magnetic pole induced in the plunger at the end nearer the solenoid will be opposite in polarity to the pole of the solenoidal winding that induced it, and the plunger will thus be attracted by the solenoidal winding. We need an expression for the induced magnetic pole of the plunger so that, if we know the characteristics of the solenoid, we can calculate the force that the solenoid will exert on the plunger. This will be worked out in Section C of this module.

**Problem 5.** A long straight wire carries a current of 5.0 A. What is the magnetic field a perpendicular distance of 8.0 cm from the wire?

**Problem 6.** A piece of wire is formed into a circular loop 8.0 cm in radius. If the current in the loop is 5.0 A, what is the magnetic field at the center of the loop? How much greater is this than the answer to Problem 5?

**Problem 7.** Many more loops 8.0 cm in radius are formed and spread out uniformly along a cylindrical core 8.0 cm in radius. Adjacent loops are 1.0 mm apart. What is the field in the core of the solenoidal winding? By what factor is this greater than the field calculated in answer to Problem 6?

### FORCE ON A CURRENT-CARRYING WIRE IN A MAGNETIC FIELD

We know that a pole of a magnet produces a magnetic field in its vicinity. This field is directed away from a north pole and toward a south pole. Its magnitude is given by Equation (7). Any other pole that is placed in the magnetic field feels a force. This force is in the direction of the field if the test pole is a north pole and in the opposite direction for a south pole. The magnitude of the force can be calculated from Equation (6) if the field is known.

A natural question to ask at this time is do forces act on a current-carrying wire that passes through a region where there is a magnetic field? To answer this you might

perform some experiments and your teacher may ask you to do so, using a *current balance*.

First, consider a straight wire which is in a magnetic field. If there is a current in the wire, a force is found to act on the wire. As shown in Figure 31, the force is perpendicular to the current and to the magnetic field lines.

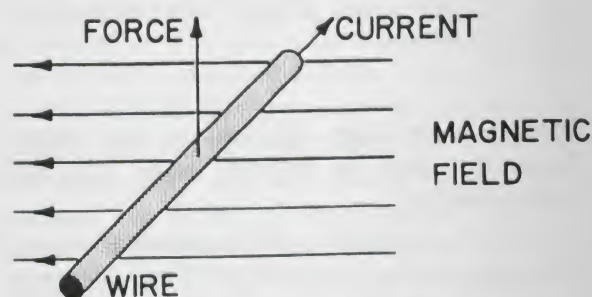


Figure 31. The force acting on a current-carrying wire in a magnetic field is perpendicular to both the wire and the field lines.

For experiments of this sort, it is possible to vary independently the three quantities involved; the current, the magnetic field, and the length of the wire which is in the magnetic field. When this is done, the following results emerge:

1. The force is proportional to the magnetic field.
2. The force is proportional to the current.
3. The force is proportional to the length of wire involved.

These results can be combined into a single proportionality:

$$F \propto BiL$$

In the SI system of units, the units of current are chosen such that this becomes the equation:

$$F = BiL \quad (12)$$

(In the somewhat more complicated case, where the wire is not perpendicular to the



magnetic field, this becomes  $F = BiL \sin \theta$ , where  $\theta$  is the angle between the field lines and the wire. This means that only the magnetic field component perpendicular to the wire ( $B \sin \theta$ ) contributes to the force on the wire.)

For experiments of this sort, it is possible to use a permanent magnet as the source of the magnetic field, but a second long straight wire is more frequently used. This is because the magnetic field of the second wire can easily be controlled by changing the current in the wire. If the current in the second wire is  $i$ , and if the two wires are parallel and a distance  $r$  apart, the magnetic field produced at the first wire by the second one is, from Equation (8):

$$B = \frac{2 ki'}{r}$$

Putting this into Equation (12) gives, for the force between two wires:

$$F = \frac{2 kii'L}{r} \quad (13)$$

This is shown in Figure 32. Experiments show that, when the two currents are in the same direction, the force on each wire is toward the other; when the currents are in opposite directions, the force on each wire is away from the other. Note also that, whatever force

acts on one wire, an equal force acts in the opposite direction on the other wire.

Physicists use Equation (13) to define the unit of current, the ampere (A). One ampere is the current required to produce a force of  $2 \times 10^{-7}$  N for each meter of length when the wires are 1 m apart.

**Example 6.** The starter cable of a car carries a current of 100 A. A heater cable running parallel to the starter cable, a distance 1 cm away from it, carries a current of 10 A. Both wires are straight and they are much longer than the distance between them (1 cm).

What is the magnetic field due to the starter current at the position of the heater cable?

Use this field to calculate the force on a 5-cm length of the heater cable due to the magnetic interaction of the two currents.

What force does a 5-cm length of the starter cable feel?

**Solution.** Equation (4) tells us that the field due to the starter wire at the position of the heater wire is:

$$\begin{aligned} B &= \frac{2 ki'}{r} \\ &= 2 \times 10^{-7} \times \frac{100}{0.01} \\ &= 2 \times 10^{-3} \text{ T} \end{aligned}$$

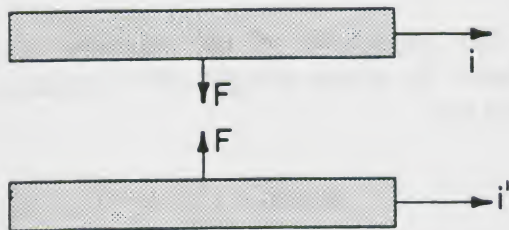


Figure 32A. Equal forces act inward on the wires.

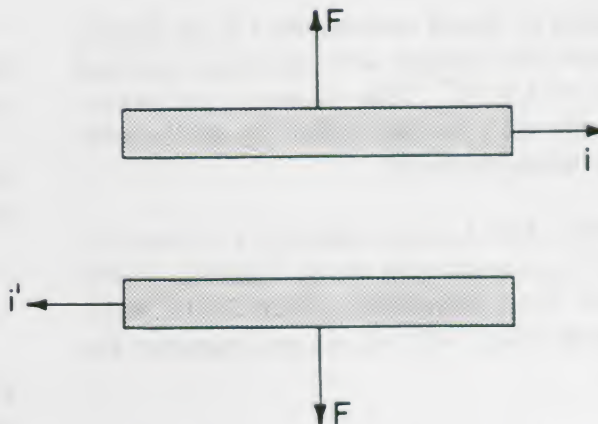


Figure 32B. Equal forces act outward on the wires.

Now that we know the field at the heater wire and the current through it, we can use Equation (12) to find the force on the heater cable.

$$\begin{aligned} F_{\text{heater wire}} &= BiL \\ &= 2 \times 10^{-3} \times 10 \times 0.05 \\ &= 1 \times 10^{-3} \text{ N} \end{aligned}$$

If we do not need to know the magnetic field at the starter cable due to the heater cable, we can use Equation (13) to compute the force on the starter cable.

$$\begin{aligned} F_{\text{starter wire}} &= \frac{2 k i i' L}{r} \\ &= 2 \times 10^{-7} \times 10 \times 100 \times \frac{0.05}{0.01} \\ &= 10^{-3} \text{ N} \end{aligned}$$

It would have been even easier to realize that this force must be the same as that calculated for the heater cable in part b.

**Problem 8.** Current is flowing through two parallel wires 0.1 m apart. One current is 4 A, and each wire feels a force toward the other of  $5 \times 10^{-6}$  N. Find the size and relative direction of the other current.

**Problem 9.** Equal currents of 3 A are flowing through two parallel wires, causing a repulsive force of  $4 \times 10^{-7}$  N/m (newtons per meter). How far apart are the wires? Are the currents in the same direction?

**Problem 10.** A wire carrying a current of 10 A is perpendicular to the direction of the earth's field, which is about 0.6 G. What magnetic force will act on one meter of the wire?

**Problem 11.** A wire of length 10 cm, carrying a current of 5 A, feels a magnetic force of

$10^{-5}$  N. Calculate the component of magnetic field perpendicular to the wire. (Saying this differently, what magnetic field perpendicular to the wire would produce this force?)

## SUMMARY

The force between two point magnetic poles,  $m$  and  $m'$ , separated by a distance  $r$  is given by the equation

$$F = \frac{kmm'}{r^2} \quad (1)$$

When two identical poles separated by one meter exert a force of  $10^{-7}$  N on each other, the strength of each pole is by definition one A·m (ampere-meter), which is the SI unit of pole strength. The value of  $k$  in SI units is  $10^{-7}$  N/A<sup>2</sup>.

The magnetic field strength  $B$  is defined as:

$$B \equiv F/m \quad (6)$$

where  $F$  is the force felt by the north test pole  $m$  due to the field  $B$ . The SI unit for  $B$  is the tesla (T), which is the same as a newton per ampere-meter (N/A·m).

The magnetic field at a distance  $r$  away from a point north pole  $m$  that is far from all other poles is given by the equation

$$B = \frac{km}{r^2} \quad (7)$$

The direction of this field is away from the pole.

The magnitude of the magnetic field produced by a long straight wire carrying a current  $i$  is

$$B = \frac{2 ki}{r} \quad (8)$$

where  $r$  is the perpendicular distance from the wire.

The magnetic field near the center of a long solenoidal winding that has  $N/L$  turns of



wire per unit length, each carrying a current  $i$ , is

$$B = \frac{4 \pi k N i}{L} \quad (10)$$

The field inside the solenoid is uniform except near the ends. On the axis at either end of the solenoidal winding, the magnetic field has half the value it has at the center.

The force on a straight piece of wire of length  $L$  carrying a current  $i$  in a magnetic field  $B$  is  $F = BiL$ , provided that the wire is perpendicular to the direction of the field. The force is perpendicular to both the length of the wire and the direction of the field lines.

Two long, parallel, current-carrying wires attract each other if the currents are in the same direction and repel each other otherwise.

## SECTION C

### THE TORQUE ON A CURRENT LOOP

The fact that a force is exerted on a current-carrying wire in a magnetic field means that a current-carrying *loop*, such as one turn of a solenoidal winding, may also experience forces and *torques* (tending to produce rotation) in a magnetic field. Let us try to calculate these for a simplified rectangular loop in a uniform field.

Figure 33 shows a view of a rectangular current loop carrying a current  $i$ . The dimensions of the rectangle are  $a$  and  $b$ . The sides of length  $b$  are perpendicular to a *uniform* (the same everywhere) magnetic field of magnitude  $B$ . The sides of length  $a$  are tilted at an angle  $\theta$  with respect to the direction of the magnetic field.

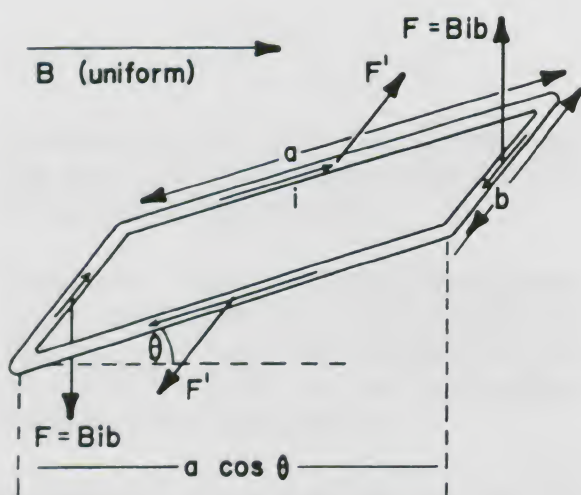


Figure 33. The torque on a current loop.

Let us first calculate the total force on the loop. This is easy because the force is zero. For every wire carrying current in one direction, there is another wire of equal length carrying the same current in the opposite direction. The magnetic field,  $B$ , is the same in magnitude and direction everywhere. Thus the magnitude of the force on one wire of an equal-length pair is the same as the force on the other member of the pair,

but since the currents in these two wires are in opposite directions, the forces on the wires also are; thus the individual forces add to a net force of zero.

The *torque* on the loop, however, is not zero. In case you have forgotten some of the essential ideas concerning torque, we include here a definition of torque and a brief discussion of those features which are important at this time.

Whenever a force acting on an object tends to rotate the object about some axis, we say there is a torque on the object. The magnitude of the torque is equal to the force multiplied by the perpendicular distance (called the *lever arm*) from the line along which the object tends to rotate. For example, if a force  $F$  is exerted at the rim of a grinding wheel of radius  $R$  in a direction tangent to the rim (see Figure 34A), the torque  $T$  on the wheel is  $FR$ .

Consider another example. A straight stick of length  $L$  is pivoted at its center and is at an angle  $\theta$  with respect to an  $x$ -axis drawn on the table top (see Figure 35). A force of magnitude  $F$  is exerted in the positive  $y$  direction at one end of the stick and another force of magnitude  $F$  is exerted in the negative  $y$  direction at the other end of the stick. (Such a pair of equal but oppositely directed forces acting on a simple object is called a *couple*.)

The stick tends to rotate about its midpoint in a counterclockwise (CCW) sense. The first force will produce a torque of magnitude  $F(\frac{1}{2}L \cos \theta)$ . (Does this agree with the definition above?) The other force also tends to rotate the stick CCW, and the torque it produces is also equal to  $F(\frac{1}{2}L \cos \theta)$ . Since the two rotations are in the same direction, we add the two torques to obtain a total torque equal to  $FL \cos \theta$ . Since  $L \cos \theta$  is simply the perpendicular distance  $d$  between the two forces, we can say the torque in this case is given by the formula

$$T = Fd \quad (14)$$



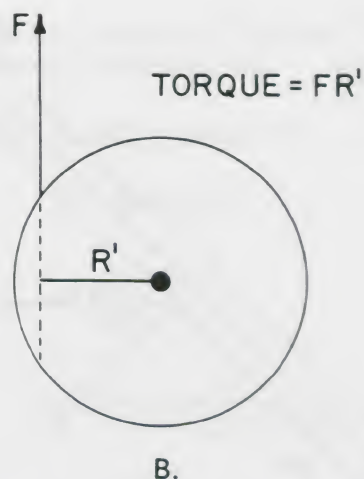
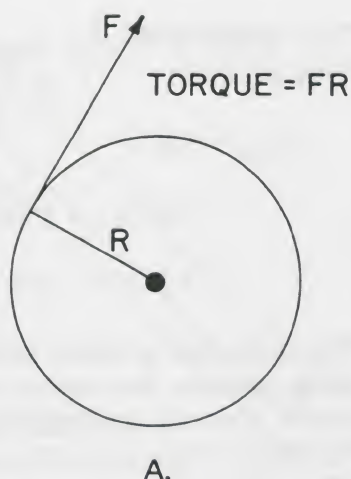


Figure 34. Torque on a grinding wheel. In each case, the torque is the force multiplied by the perpendicular distance to the axis of rotation.

(Note that, if you imagine the pivot to be anywhere else along the stick, you obtain the same formula for the total torque.)

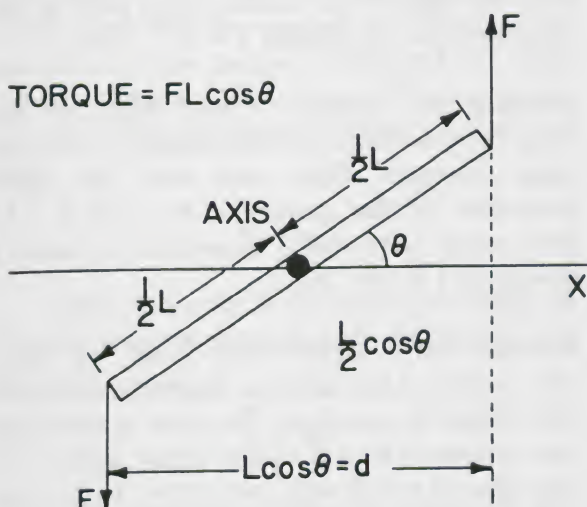


Figure 35. Torque on a stick.

Let us now return to the rectangular loop pictured in Figure 33. The two wires of length  $a$  are not perpendicular to the magnetic field. Nevertheless, each has a component perpendicular to  $B$ , and thus there is a force on each of these wires. Each of these is perpendicular to the page and, because the currents are opposite, one force is out of the page and one into it. We need not calculate these forces because not only are they equal

and opposite in direction, and thus produce no net force, but they lie along the same line and thus produce no net torque either. The forces on the two wires of length  $b$  do produce a torque. According to Equation (12), the magnitude of each force is  $Bib$ . The directions of the two forces are opposite, but the lines of action are not the same, thus they form a couple. In fact, the distance between the two forces,  $d$ , is  $(a \cos \theta)$ ; thus, according to Equation (14) the net torque on the loop is

$$T = Fd = Biba \cos \theta$$

The area of the loop,  $A$ , is equal to  $ab$ , thus the torque can be written as

$$T = BiA \cos \theta$$

If we want to be able to specify the orientation of the area  $A$  in space, we can think of the area  $A$  as having a direction associated with it that is perpendicular to the plane containing the loop. A line perpendicular to the plane is called the *normal* to the plane. The angle  $\alpha$  that this line makes with the magnetic field is the complement of  $\theta$  (see Figure 36); thus  $\sin \alpha = \cos \theta$ . Therefore, the torque equation becomes

$$T = BiA \sin \alpha \quad (15)$$

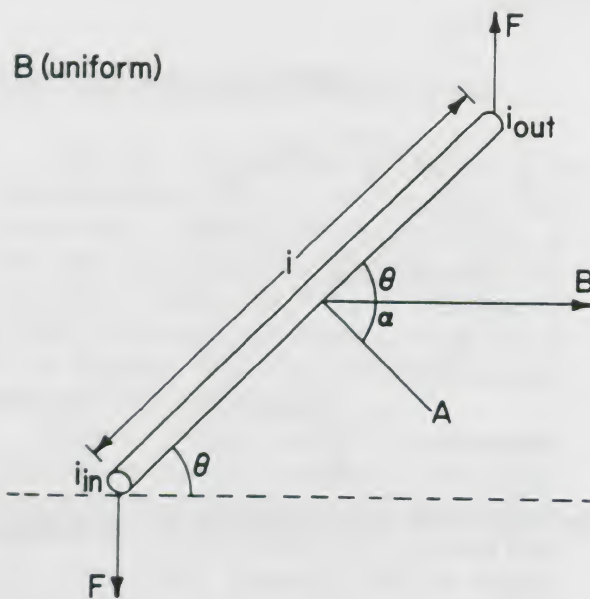


Figure 36. Cross section of current loop. At the top of the loop the current is out of the drawing; at the bottom it is in. The normal, marked  $A$ , is perpendicular to the plane of the loop.

This torque tends to rotate the loop so as to make  $\alpha$ , and thus the torque, zero. It turns out that this same formula applies to a loop of any shape whatever, although it is more difficult to prove it for other shapes.

**Example 7.** A common electric meter consists of a coil of wire mounted on bearings and located in the magnetic field of an iron magnet. In such a meter the coil might be in a uniform magnetic field  $B = 0.1$  T. The normal to the coil makes an angle of  $30^\circ$  with the magnetic field and the rectangular coil has an area  $1 \text{ cm} \times 1.5 \text{ cm} = 1.5 \times 10^{-4} \text{ m}^2$ .

What is the torque in newton-meters (Nm) on each turn of the coil when the current is  $0.2 \text{ A}$ ?

What is the total torque on a coil of 500 turns for the  $0.2\text{-A}$  current?

**Solution.** From Equation (15)

$$\begin{aligned} T &= BiA \sin \alpha \\ &= 0.1 \times 0.2 \times 1.5 \times 10^{-4} \times 0.5 \\ &= 1.5 \times 10^{-6} \text{ Nm} \end{aligned}$$

This torque is exerted on each turn, so 500 turns will have:

$$\begin{aligned} T_{\text{net}} &= 500 T \\ &= 500 \times 1.5 \times 10^{-6} \\ &= 7.5 \times 10^{-4} \text{ Nm} \end{aligned}$$

The meter has a spring attached to the coil which opposes this torque produced by the current. You can see from Equation (15) that the torque depends on the current in the coil. The larger the current, the larger the torque. Therefore, when more current flows, the coil pushes harder against the spring and reads higher on the scale.

**Problem 12.** A loop of wire of area  $0.05 \text{ m}^2$  is placed in a magnetic field  $B = 0.01 \text{ T}$ . The normal to the loop makes an angle of  $30^\circ$  with the field. What current must flow in the loop to cause a torque of  $1 \times 10^{-3} \text{ Nm}$ ?

**Problem 13.** Consider a coil made of the same wire as the loop in Problem 12, in the same magnetic field, and with the same geometry. If the current in the coil is  $1 \text{ A}$ , how many turns would be needed to cause a torque of  $1 \times 10^{-3} \text{ Nm}$ ?

**Problem 14.** A square wire loop,  $1 \text{ m}$  on a side, is placed in a uniform magnetic field and the torque is measured. The wire is unfolded and reformed into a square shape again, but this time as a coil with *two* turns. If the new coil is put in the original field at the same angle as the  $1\text{-m}$  loop, what torque will act on the coil?

### The Torque on a Bar Magnet

Now consider a bar magnet in a uniform magnetic field. You know that the magnet tends to rotate in the field (like the compass needle in Section A); thus there must be a torque on it. Let us calculate this torque. Refer to Figure 37. There you see a bar magnet of length  $L$  in a uniform magnetic



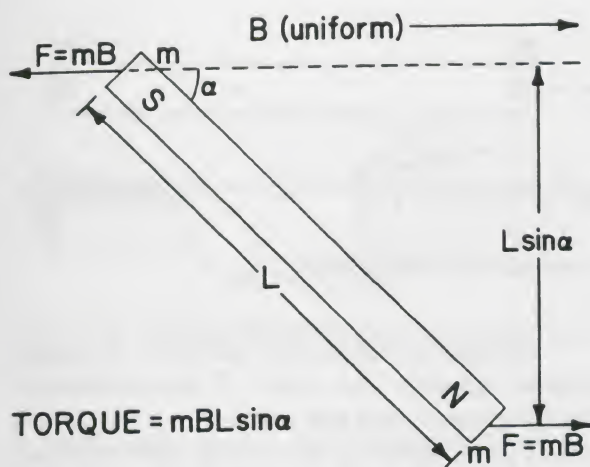


Figure 37. Torque on a bar magnet.

field  $B$ . The magnet makes an angle  $\alpha$  with the direction of the field. The magnet has a north pole on one end and a south pole on the other. Each pole has a strength  $m$ . The north pole of the magnet is pulled in the direction of the magnetic field and the south pole in the opposite direction. The magnitude of each of these forces is given by Equation (6).

$$F = mB$$

Now it is easy to calculate the torque on the bar magnet. According to Equation (14), it is

$$T = Fd$$

From the geometry shown in Figure 37,  $d = L \sin \alpha$ . Thus we arrive at the formula for the torque on a bar magnet in a magnetic field:

$$T = BmL \sin \alpha \quad (16)$$

The torque always acts in such a direction that  $\alpha$  tends toward zero; thus the magnet aligns itself with the field.

**Example 8.** A bar magnet of length 10 cm is placed in a uniform magnetic field  $B = 0.1$  T at an angle of  $45^\circ$  with the field. The magnet

experiences a torque of 0.2 Nm. What is the pole strength of the magnet?

**Solution.** From Equation (16)

$$T = BmL \sin \alpha$$

$$\begin{aligned} m &= \frac{T}{BL \sin \alpha} \\ &= \frac{0.2}{0.1 \times 0.1 \times .707} \\ &= 28.3 \text{ A}\cdot\text{m} \end{aligned}$$

**Problem 15.** What is the force on each of the poles of the magnet in Example 8? Show that the torques from the forces add to give the observed torque of 0.2 Nm experienced by the magnet in Example 8.

**Problem 16.** A bar magnet of pole strength 10 A·m and length 0.1 m is placed in a uniform magnetic field perpendicular to the field lines. If the torque on the magnet is 0.5 Nm, what is the magnetic field? Could a device which measures the torque on a bar magnet be used to measure magnetic fields?

### THE FORCE BETWEEN A COIL AND A BAR MAGNET

Now compare Equations (15) and (16). They are very similar. In fact, they tell us that, if a current loop and a bar magnet are both placed in a uniform magnetic field, and oriented so that the angles  $\alpha$  in Figures 36 and 37 are the same, and if the product  $iA$  for the loop equals the product  $mL$  for the magnet, the two objects experience the same torque. That is, as far as their magnetic properties are concerned, the two objects are indistinguishable. We will use this fact to calculate the force that a solenoidal winding exerts on a magnetized plunger.

A solenoidal winding and a bar magnet are shown in Figure 38. The bar magnet has poles of strength  $m'$  separated by a distance

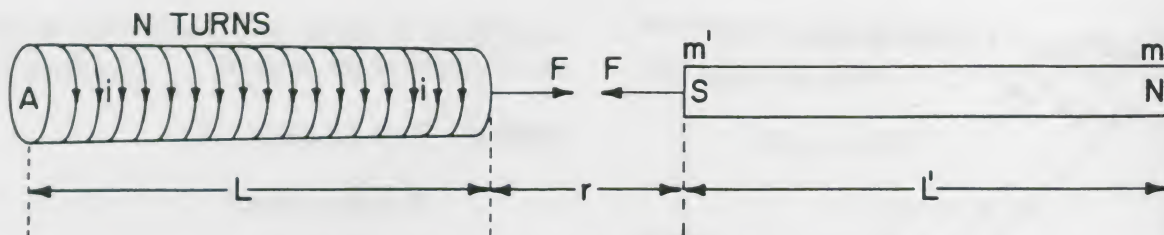


Figure 38. Attraction between a solenoid and a bar magnet.

$L'$ . The solenoidal winding carries a current  $i$  through  $N$  loops, each of area  $A$ . According to the previous paragraph, if each loop were replaced by a small bar magnet of length  $a$  and pole strength  $m$ , such that  $iA = ma$ , no one could tell the difference so far as the magnetic behavior is concerned. How long should each of these small bar magnets be? Long enough so that when they are placed end to end, they will be as long as the solenoidal winding; that is,  $Na = L$ . What is the net effect of putting small magnets end to end? The south pole of one short magnet cancels the effect of a north pole of the next, and this happens all the way to the ends of the line of magnets. Thus, aligning  $N$  short bar magnets of pole strength  $m$  and length  $a$  creates a single bar magnet of length  $L = Na$  and pole strength  $m$ . Thus we have only the problem of finding the force between two bar magnets placed in the geometrical arrangement shown in Figure 38. One magnet has a pole strength  $m'$  and the other "magnet" has a pole strength  $m$ . The pole strength  $m$  is related to the current, area, length, and number of loops of the solenoidal winding by the equation

$$m = \frac{iA}{a}$$

Using  $a = L/N$ , this becomes

$$m = \frac{NiA}{L} \quad (17)$$

The force can now be found by using Coulomb's Law, Equation (1).

Before we tackle this problem, we must decide whether the pole of the solenoidal winding nearer the bar magnet is a north or a south pole. Refer to the results you obtained in Experiments A-3 and B-1. These should convince you that the following rule is correct. If you place your right hand around the solenoid so that your fingers curl around it pointing in the same direction as the current is travelling through the loops, your thumb will point in the direction of the north pole. This is indicated in Figure 39. Applying the rule to the solenoid of Figure 38 places the north pole to the right.

The problem of finding the net force between the solenoid and the magnet is complicated by the fact that each pole of the magnet interacts with each pole of the solenoid, and every interacting pair involves a different separation distance. We will simplify this analysis by assuming that the lengths  $L$  and  $L'$  are large compared to the separation of the near poles,  $r$ . Under these conditions, we need only consider the force between the pair of poles that are closest together and we may ignore the other forces. According to Equation (1), this force is given by

$$F = k \frac{mm'}{r^2}$$

Or, using Equation (17):

$$F = \frac{k Ni Am'}{Lr^2} \quad (18)$$

In deriving Equation (18), we have assumed that both poles are points; thus the equation





Figure 39. Finding the north pole of a solenoidal winding.

is valid only if the cross-sectional areas of both the permanent magnet and the solenoidal winding are small compared to their lengths.

### THE FORCE ON AN INDUCED POLE IN A MAGNETIC FIELD

Up to now, we have been thinking of the solenoid plunger as a permanent bar magnet with a pole strength  $m'$ . But, in reality, the plunger is usually a piece of soft iron which is not a magnet at all. It becomes a magnet (is magnetized) when it is placed in a magnetic field. Obviously, if we wish to know the force exerted on this *induced magnet* by our solenoidal winding, we must know the value of the *induced pole strength*,  $m'$ . The induced pole strength depends strongly on a property of the iron called magnetic *permeability*. Soft iron has a large permeability, which means that the induced pole strength for a plunger of this material is larger than for other materials under similar conditions. This is why the solenoid designer chooses soft iron for the plunger. In practice, solenoids are

likely to be designed like the one shown in Figure 40.

The strength of the induced pole also depends on the magnetic field in which it finds itself. Thus, as the plunger is moved from place to place, or as the current in the solenoidal winding changes, the magnetic field that magnetizes the iron plunger changes. Therefore, the induced pole strength of the plunger changes. How can we make sense of this situation? Consider the two variations one by one.

First, imagine that the current in the solenoidal winding is fixed. We want to discover how the force of attraction changes as we move the plunger from one place to another in the magnetic field produced by the solenoidal winding. The best way to find out is to do an experiment, and that is precisely what you will do in Experiment C-1.

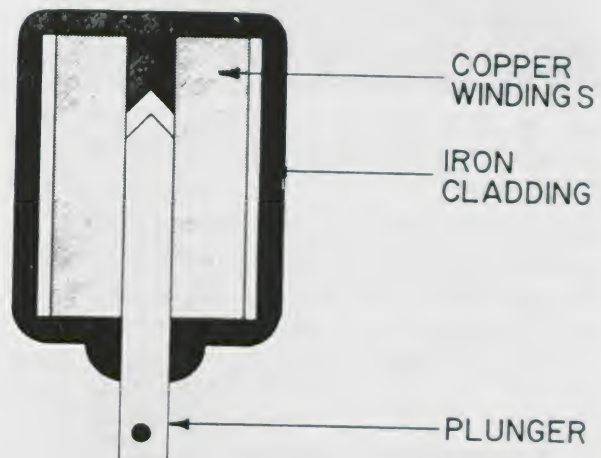


Figure 40. Cross section of a solenoid.

## EXPERIMENT C-1. Force versus Plunger Position

In this experiment you will study the force versus length-of-insertion characteristics of a solenoid with constant current.

### Procedure

1. Set up the apparatus as shown in Figure 41. The weight-hanger and weights should just take up the slack in the linkages between the plunger, the lever, and the spring balance. The balance should read zero when there is no current in the solenoidal winding.

**CAUTION:** Do not touch exposed wires on terminals!



Figure 41.

2. Withdraw the plunger as much as possible. Measure the length of plunger that is visible when it is fully withdrawn.
3. Close the electrical circuit and set the current at 2.0 A.

4. Move the scale so that it pulls the plunger just to the fully withdrawn position. Read the force in newtons from the scale and record it. This force corresponds to minimum insertion.
5. Now move the scale so that the insertion is a millimeter or two more than minimum. Measure the length of plunger visible and record the amount of insertion above minimum. Read and record the corresponding force.
6. Continue this process until the plunger is fully inserted. You should have recorded at least six sets of readings.
7. Plot a graph of force in newtons versus the insertion of the plunger above the minimum insertion, in millimeters.

### Questions

1. The graph of Figure 42 shows the force on a plunger which can be completely removed from and fully inserted into the solenoidal winding. Does your graph bear any resemblance to that shown in Figure 42? Which portion of Figure 42 corresponds to your graph?

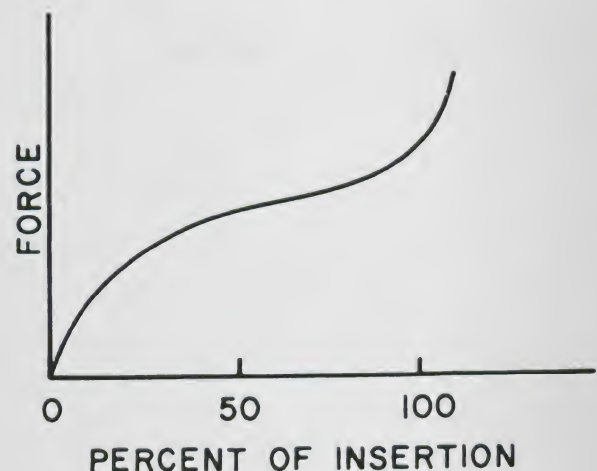


Figure 42.



2. What is the function of the lever in the experiment?
3. Can you think of an application for a solenoid in which the force should be small when the insertion is small but larger when the insertion is complete?

Let us now see if we can understand the results of Experiment C-1 and the graph of Figure 42. In Experiment A-3, you observed that, when the plunger is in a weak magnetic field, the induced pole strength is small. When the plunger is moved closer to the source of the field, and therefore, into a stronger field, the strength of the induced pole is much larger. If these comparisons were made quantitatively, the results would show that the induced pole strength  $m'$  is proportional to the magnetic field the plunger is in, at least for small values of  $B$ .

We know that the magnetic field produced at the center of the solenoid is given by Equation (10), and at the end of the solenoid the field is given as Equation (11). In other words, as one moves from the center to the end of the solenoidal winding, the magnetic field decreases by a factor of two. This means that the induced pole strength in the plunger should change approximately by a factor of two as the end of the plunger is moved from the middle of the winding to a position near one end. Looking at Equation (6), since both  $m'$  and  $B$  change by factors of two, we may expect the force to change by a factor of four over this range of plunger movement.

How does the force vary with the position of the plunger for positions between these two extremes? This is hard to predict with any precision, using only the simple theory we have considered above. We have neglected the fact that in practice both the solenoidal winding and the plunger are quite short. So the approximation that  $r$  is small compared to the length, which we used to derive the dependence of  $F$  on  $r$  given by Equation (18), is not valid for a real solenoid. In fact, the plunger of a real solenoid is never completely withdrawn from the core of the winding in the way we have suggested in Figure 38. Moreover, neither the pole of the plunger nor the equivalent pole of the solenoidal winding behaves as a point pole, so the application of Equation (1) to our solenoid would require some hard calculations, and we will not attempt them here. Finally, real solenoids have iron cladding around the winding, and this tends to increase the field in and around the core, as you observed in the last part of Experiment A-5. The cladding also modifies the shape of the field lines inside the solenoidal winding. The net result is a graph like that of Figure 42 for the force on the plunger versus percent of insertion into the coil.

Having examined the force on the plunger as it is inserted into the solenoid, we now wish to examine the variation in the force as the current in the solenoid is changed. You may begin this study by doing Experiment C-2.



## EXPERIMENT C-2. Force on a Solenoid Plunger as a Function of Current in the Winding

In this experiment you will study the force versus current characteristics of a solenoid with constant insertion of the plunger.

### Procedure

**CAUTION:** Large currents are present in this experiment. Do not touch terminals or wires when the current is on.

1. Set up the apparatus as described at the beginning of Experiment C-1.
2. Move the scale so that the solenoid plunger protrudes about 10 mm when the current is zero.
3. Adjust the knob on the power supply so that 1 A flows in the solenoidal winding. Read the force on the scale after moving the scale so that the plunger protrudes the same amount as in step 2.
4. Increase the current by half-ampere steps and repeat the force readings called for in step 3. Go to as high a value of current as your equipment will stand. (Check with your instructor.) Always make sure before each scale reading that the plunger protrudes the same amount.
5. Subtract the zero-current scale reading from each force reading. Use the results to plot force versus current, with force in newtons as the ordinate and current in amperes as the abscissa.
6. Repeat steps 3, 4, and 5 using a plunger protrusion of 14 or 15 mm.

### Questions

1. Why is the zero-current scale reading subtracted from each force reading in step 5?
2. Does it matter that we plot the force

versus the current without correcting for the action of the lever?

3. Do your graphs rise rapidly for small  $i$  and more slowly for large  $i$ ? You will see that this is predicted by theory.

Let us now examine a theory for how the force on a fully inserted solenoid plunger varies as the current in the solenoidal winding varies. Note that in all the formulas we have written for the magnetic fields produced by currents, such as Equations (8), (9), (10), and (11), the magnetic field is proportional to the current in the circuit. If the current in the solenoid is doubled, the magnetic field everywhere is also doubled. Thus, wherever the plunger is,  $B$  at that point is proportional to  $i$ . If it is also true that the induced pole strength  $m'$  for an originally unmagnetized piece of iron is proportional to the field  $B$  it finds itself in, then  $m'$  must be proportional to the inducing current  $i$ . It turns out that this relationship does hold for very small currents. Thus the force between the solenoidal winding and the plunger, given by Equation (6) where  $B$  is the field of the solenoidal winding, should turn out to be proportional to  $i^2$ . That is,

$$F = Ki^2 \quad (\text{for small } i) \quad (19)$$

where  $K$  is a constant.

When the plunger is in a very strong magnetic field, the magnetic behavior of the plunger is very different. Under this condition, the induced pole strength  $m'$  does not increase any more as the magnetic field increases. One says that the induced pole strength has reached *saturation*. There is a "model" for the internal structure of iron which attempts to explain both of these behaviors. It suggests that the process of magnetization involves the alignment of tiny current loops in the atoms of the material. As long as only a few are aligned, then doubling the flux density will double the number



aligned and thus double  $m'$ . However, once all the atomic current loops are aligned, there can be no further increase in  $m'$ . When this latter condition is reached, the iron is said to be saturated. The maximum value of  $m'$  is a characteristic of the material; we will call it  $M'$ . According to Equation (6), the force on  $M'$  is proportional to the  $B$  field. But the  $B$  field, in turn, is proportional to the current in the winding. Thus, above saturation, the force depends on the current as:

$$F = F_s + K'i \quad (20)$$

where  $F_s$  is the force at saturation and  $K'$  is a constant different from  $K$ .

From Equations (19) and (20), we expect that:  $F$  varies with  $i^2$  for small  $i$ ; then  $F$  grows more slowly with  $i$  as  $i$  increases through intermediate values; finally  $F$  increases in proportion to the increase in  $i$  for large currents. This is shown roughly in the graph of Figure 43. See if your results are consistent with this fairly complicated prediction.

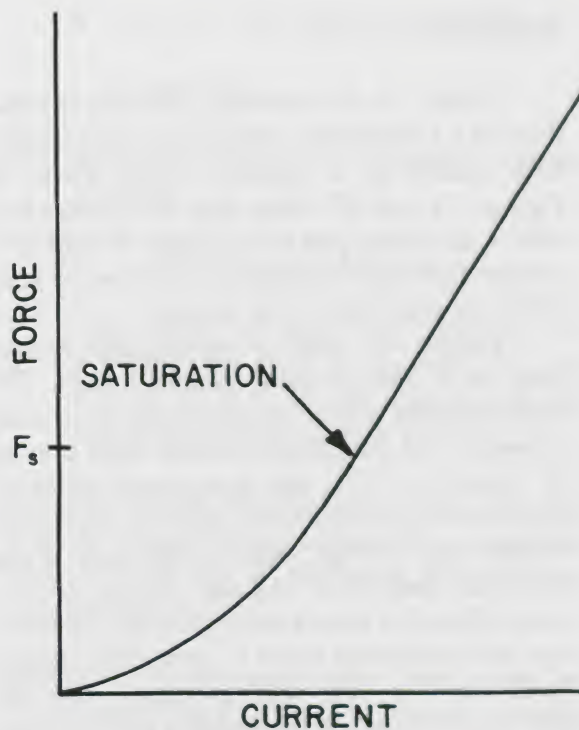


Figure 43.

## MAGNETIC FLUX

Earlier in this section, you learned that there is a torque that tends to rotate a current loop placed in a magnetic field. Refer to Figures 33 and 35. Note that the torque is in such a direction that the number of field lines passing through the loop increases as the loop rotates in response to the torque.

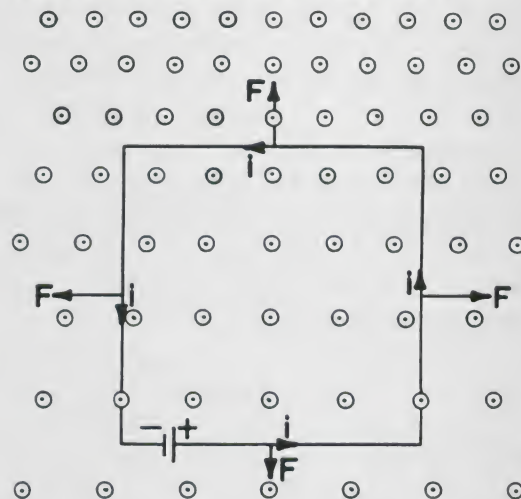
Figure 44 shows a rectangular current loop in a *non-uniform* magnetic field. The force on each side of the rectangle is directed outward. The magnitude of each force is given by Equation (12). The horizontal forces on the two vertical sides are equal in magnitude, because the average value of the field is the same for each wire and the lengths are the same. Since the forces are oppositely directed, the net horizontal force is zero. The upward force on the top wire is larger than the downward force on the bottom wire, because the magnetic field is larger at the top of the rectangle. The net vertical force is upward. This force tends to move the loop into a region where the magnetic field is large. Thus, if the loop moves in response to this force, the number of field lines through the loop will increase.

In each case, when only magnetic forces act, a current loop moves in a way that results in an increase in the number of magnetic field lines passing through the loop. This suggests that the number of magnetic field lines that pass through a given area might be a useful number.

Of course, the number of lines in a diagram depends only on how many lines we *choose* to draw. However, once that choice is made, then the density of lines (the number per unit area) will always be greater in regions where the magnetic field is large. Then the number that pass through any area we wish to consider must depend on both the magnitude of the field  $B$  and the area  $A$ . (It also depends on whether the area is tilted with respect to the lines.) An easy way to compute a useable number is to calculate the product of  $B \times A$ . The product is arbitrarily defined to be the magnetic flux:

$$\Phi = BA \quad (21)$$

## STRONG MAGNETIC FIELD OUT



## WEAK MAGNETIC FIELD OUT

Figure 44. A rectangular current loop in a non-uniform magnetic field. The field lines are all perpendicular to the plane of the loop and out of the paper, but the field is stronger at the top of the figure.

In SI units, where  $B$  is in teslas and  $A$  is in square meters,  $\Phi$  is in webers (Wb). Considering this definition, it is understandable why  $B$  is often referred to as the *flux density* and why units of  $B$  were once called *webers per square meter*. Notice that a greater flux density means a greater magnetic field. This means that in regions of greater magnetic field, the field lines are closer together. This can be seen by looking at the magnetic field lines of a bar magnet; near the poles, where the field is strongest, the field lines are bunched most closely.

The concept of flux is useful. It is especially important when the flux through a conducting circuit is changed in any way, such as changing the current in a solenoid.

**Example 9.** A wire loop of dimensions 10 cm  $\times$  40 cm is placed in a magnetic field so that it is perpendicular to the field lines.  $B = 0.2$  T.

1. What is the magnetic flux through the loop?



- If the coil is rotated so that it makes an angle of  $60^\circ$  with the field, as shown in Figure 45, what is the flux through the loop?
- Express the flux through the loop for any angle  $\theta$  that the loop makes with the field. For what angle is the flux greatest? Least?

**Solution.**

- From Equation (21), putting the lengths into meters:

$$\Phi = BA = 0.2 \times 0.1 \times 0.4 = 0.008 \text{ Wb}$$

- When the coil is rotated, the number of magnetic field lines passing through it is the same as the number which would pass through a coil whose length is  $L \sin 60^\circ$ . This can be seen from the diagram of Figure 46. The sides of the coil which are perpendicular to the paper are still perpendicular to the magnetic field lines, so the effective width is not changed. The effective area of the coil, when it has been rotated  $60^\circ$ , is therefore:

$$A_{\text{eff}} = L_{\text{eff}}w = L \sin 60^\circ w = A \sin 60^\circ$$

The magnetic flux through the loop is:

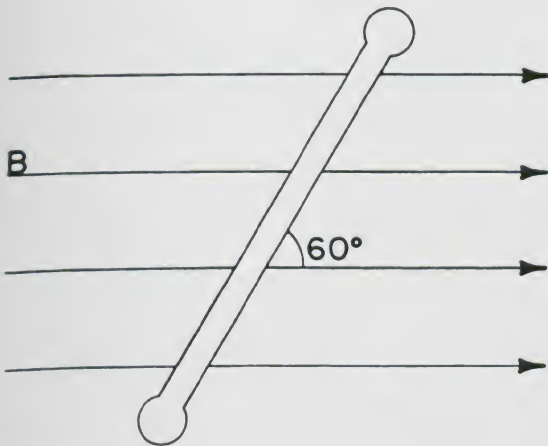


Figure 45. A 10-cm by 40-cm current loop in a magnetic field. The 10-cm sides are perpendicular to the paper.

$$\begin{aligned}\Phi &= BA_{\text{eff}} = 0.2 \times 0.1 \times 0.4 \times 0.87 \\ &= 0.007 \text{ Wb}\end{aligned}$$

- From the discussion in part b, one can see that

$$\Phi = BA_{\text{eff}} = BA \sin \theta$$

This tells us that for  $\sin \theta = 1$ , or  $\theta = 90^\circ$ , the flux is greatest. For  $\sin \theta = 0$ , or  $\theta = 0^\circ$ , the flux is zero because the effective area is zero.

**Problem 17.** A circular wire loop of radius 1 m is placed perpendicular to a magnetic field  $B = 2 \times 10^{-2}$  T. Calculate the magnetic flux through it.

**Problem 18.** A rectangular wire loop of dimensions 1 m  $\times$  0.5 m is placed at an angle of  $45^\circ$  to a field  $B = 1$  T. What is the magnetic flux through it?

**Problem 19.** Consider a loop of flexible wire. This loop is placed in a uniform magnetic field perpendicular to the plane of the loop as shown, while a current is passing through the wire.

- Show that the wire will move so as to increase the flux through the loop.

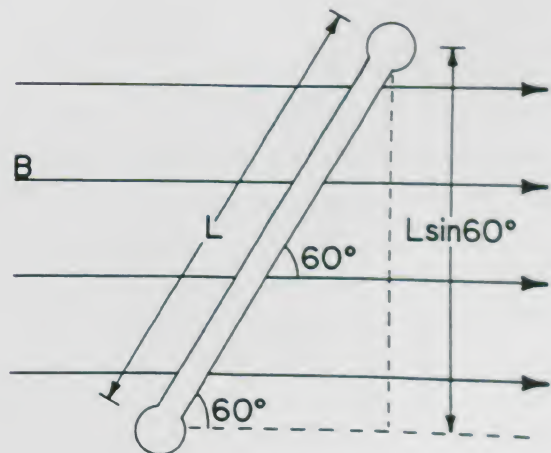


Figure 46.

2. If the current were reversed, does it look as if the flux through the loop will be increased by the wire's motion? How will the wire move so that the flux will be increased?

### Question

Can you think of some applications around the house or the car where a solenoid would be a work saver?

### SUMMARY

A current loop in a magnetic field experiences a torque that tends to rotate the loop so that the plane of the loop is perpendicular to the field lines.

A bar magnet in a magnetic field feels a torque that tends to align the magnet with the field lines.

A current loop and a bar magnet behave the same way in a magnetic field provided that the product  $iA$  for the loop is equal to the product  $mL$  for the bar magnet.

Current-loop magnets exert forces on bar magnets.

The strength of the magnetic poles induced on the ends of a previously unmagnetized soft iron bar, such as a solenoid plunger,

is proportional to the magnetic field that causes the poles to be induced, at least for small fields. For large fields the iron bar may reach saturation, a condition where the induced pole strength is a maximum for that particular bar.

The force exerted on a solenoid plunger by the field of a solenoidal winding with a fixed current varies as the plunger moves into the solenoid core. In real solenoids this force increases as the plunger is inserted deeper into the core.

For small currents in the solenoidal windings, the induced pole strength of the solenoid plunger increases in proportion to the current and the force on the plunger increases as the square of the current.

When the inducing field is very large, the induced pole strength on a solenoid plunger reaches a maximum value (becomes saturated). For currents above those that produce saturation, the force on the solenoid plunger increases as the first power of the current.

Magnetic flux  $\Phi$  through an area  $A$  is equal to the product of the area times the component of magnetic field perpendicular to the plane,  $B_p$ :

$$\Phi = B_p A$$





